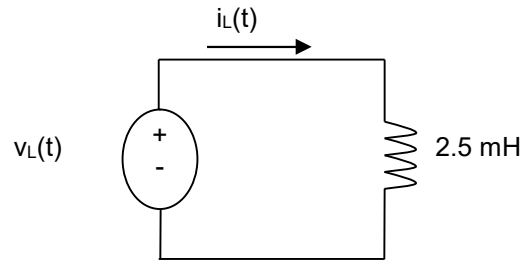


# Fundamentals of Electrical Circuits - Chapter 6

1S. The current in the 2.5 mH Inductor in the following figure is known to be 0A for  $t < 0$ . The inductor voltage for  $t \geq 0$  is given by the expression:

$$v_L(t) = \begin{cases} 3 e^{-4t} \text{ mV} & 0 \leq t \leq 2 \text{ s} \\ -3 e^{-4(t-2)} \text{ mV} & 2 \text{ s} \leq t < \infty \end{cases}$$



Sketch  $v_L(t)$  and  $i_L(t)$  for  $0 \leq t < \infty$

**Solution:**

$$i_L(t) = \frac{1}{L} \int_{t_0}^t V dt + i(t_0)$$

$$0 \leq t \leq 2$$

$$i_L(t) = \frac{1}{2.5 * 10^{-3}} \int_0^t 3 * 10^{-3} e^{-4\tau} d\tau + i_L(0)$$

$$i_L(t) = \frac{3}{2.5} \int_0^t e^{-4\tau} d\tau + 0 = \frac{3}{2.5} * \frac{1}{-4} e^{-4\tau} \Big|_0^t$$

$$i_L(t) = -.3e^{-4t} + .3 \quad \{\text{eq1}\}$$

$$2 \leq t < \infty$$

$$i_L(t) = \frac{1}{2.5 * 10^{-3}} \int_2^t -3 * 10^{-3} e^{-4(\tau-2)} d\tau + i_L(2^-)$$

$$i_L(t) = \frac{-3}{2.5} \int_2^t e^{-4(\tau-2)} d\tau + i_L(2^-)$$

$$i_L(t) = \frac{-3}{2.5} * \frac{1}{-4} e^{-4(\tau-2)} \Big|_2^t + i_L(2^-) \quad \{\text{eq 2}\}$$

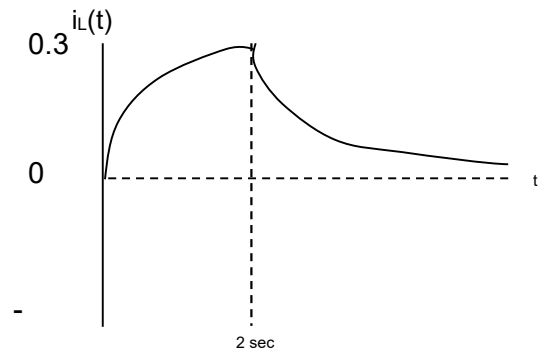
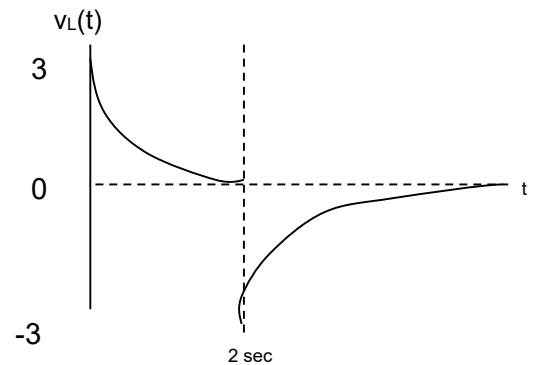
since initial current cannot change instantly then we can use  $i_L(2^-)$  from equation {eq 1}:

$$i_L(2^-) = -.3e^{-4(2)} + .3$$

Plug value in  $i_L(2^-)$  in {eq 2}

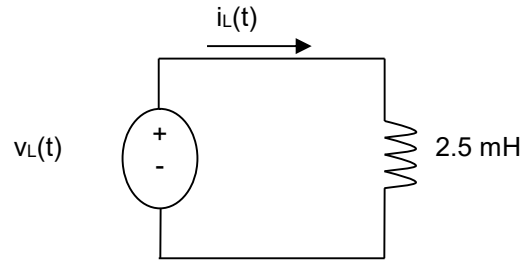
$$i_L(t) = .3e^{-4(t-2)} - .3 - .3e^{-8} + .3$$

$$i_L(t) = .3e^{-4(t-2)} - .3e^{-8}$$



1U. The current in the 2.5 mH Inductor in the following figure is known to be 0A for  $t < 0$ . The inductor voltage for  $t \geq 0$  is given by the expression:

$$v_L(t) = \begin{cases} 7 e^{-9t} \text{ mV} & 0 \leq t \leq 6 \text{ s} \\ 4 e^{-8(t-4)} \text{ mV} & 6 \text{ s} \leq t \leq \infty \end{cases}$$



Sketch  $v_L(t)$  and  $i_L(t)$  for  $0 \leq t \leq \infty$

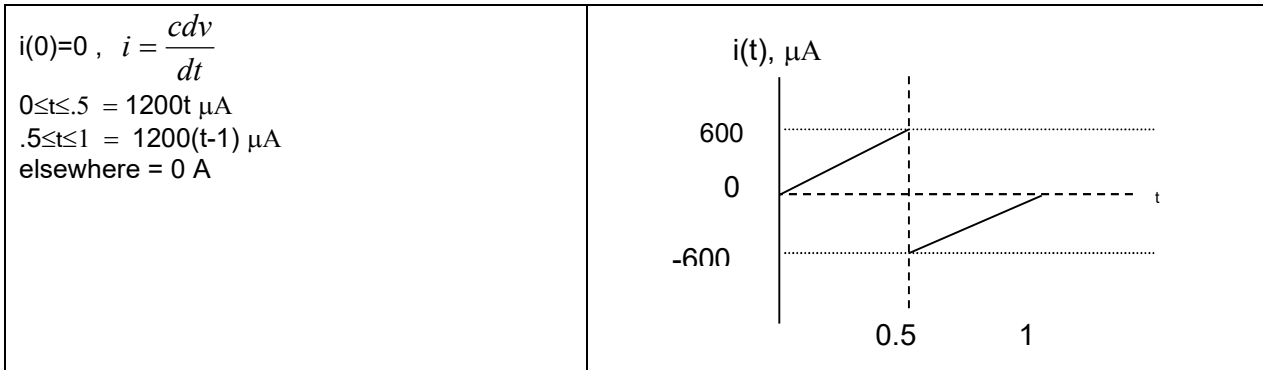
**Solution:**

2S. A 20  $\mu\text{F}$  capacitor is subjected to a voltage pulse having a duration of one seconds. The pulse is described by the following equations:

$$v_C(t) = \begin{cases} 30 t^2 \text{ V} & 0 \leq t \leq 0.5 \text{ s} \\ 30(t-1)^2 \text{ V} & 0.5 \text{ s} \leq t \leq 1.0 \text{ s} \\ 0 & \text{elsewhere} \end{cases}$$

Sketch the current pulse that exists in the capacitor during the 0 to 1 second interval.

**Solution:**



2U. A 10  $\mu\text{F}$  capacitor is subjected to a voltage pulse having a duration of two seconds. The pulse is described by the following equations:

$$v_C(t) = \begin{cases} 10 t^2 \text{ V} & 0 \leq t \leq 1.5 \text{ s} \\ 20(t-2)^2 \text{ V} & 1.5 \text{ s} \leq t \leq 2.0 \text{ s} \\ 0 & \text{elsewhere} \end{cases}$$

Sketch the current pulse that exists in the capacitor from  $t=0$  to 2 seconds.

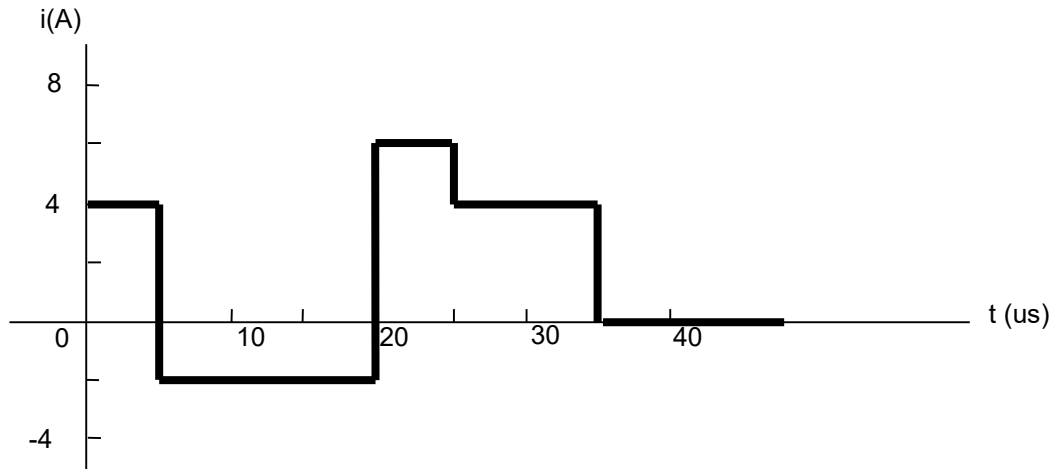
**Solution:**

3S. The rectangular-shaped current pulse shown in the following figure is applied to a 5  $\mu\text{F}$  capacitor. The initial voltage on the capacitor is 12 V drop in the reference direction of the current. Assume the passive sign convention. Derive the expression for the capacitor voltage for the time intervals in (a)-(e).

- $0 \leq t \leq 5 \text{ } \mu\text{s}$
- $5 \text{ } \mu\text{s} \leq t \leq 20 \text{ } \mu\text{s}$
- $20 \text{ } \mu\text{s} \leq t \leq 25 \text{ } \mu\text{s}$
- $25 \text{ } \mu\text{s} \leq t \leq 35 \text{ } \mu\text{s}$

e)  $35 \mu\text{s} \leq t \leq \infty$

f) Sketch  $v(t)$  over the interval a)  $0 \mu\text{s} \leq t \leq 300 \mu\text{s}$ .



**Solution:**

$$c=5 \mu\text{F}$$

$$V(0)=12\text{V}$$

$$V = \frac{1}{c} \int_{t_0}^t i(t)dt + V(0)$$

a)  $0 \leq t \leq 5 \mu\text{Sec}$

$$V(t) = 200,000 \int_0^t 4dt + 12V = 200,000(4t) + 12V = 8 \cdot 10^5 t + 12V$$

$$@ t=5 \cdot 10^{-6} \quad v(t)=16V$$

b)  $5 \mu\text{Sec} \leq t \leq 20 \mu\text{Sec}$

$$V(t) = 2 \cdot 10^5 \int_{5 \cdot 10^{-6}}^t -2dt + V(5 \cdot 10^{-6}) = -4 \cdot 10^5 [t - 5 \cdot 10^{-6}] + 16 = -4 \cdot 10^5 t + 18$$

$$@ t=20 \cdot 10^{-6} \quad v(t)=10V$$

c)  $20 \mu\text{Sec} \leq t \leq 25 \mu\text{Sec}$

$$V(t) = 2 \cdot 10^5 \int_{20 \cdot 10^{-6}}^t 6dt + V(20 \cdot 10^{-6}) = 12 \cdot 10^5 (t - 2 \cdot 10^{-5}) + 10 = 12 \cdot 10^5 t - 14$$

$$@ t=25 \cdot 10^{-6}, \quad V(25 \cdot 10^{-6})=16V$$

d)  $25 \mu\text{Sec} \leq t \leq 35 \mu\text{Sec}$

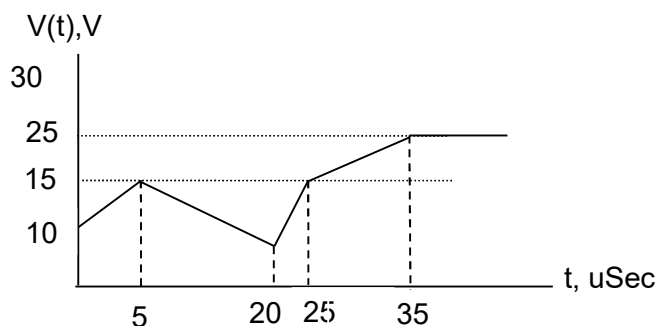
$$V(t) = 2 \cdot 10^5 \int_{25 \cdot 10^{-6}}^t 4dt + V(25 \cdot 10^{-6}) = 8 \cdot 10^5 (t - 25 \cdot 10^{-6}) + 16 = 8 \cdot 10^5 t - 4$$

$$@ t=35 \cdot 10^{-6}, \quad V(35 \cdot 10^{-6}) = 24V$$

e)  $35 \mu\text{Sec} \leq t \leq \infty$

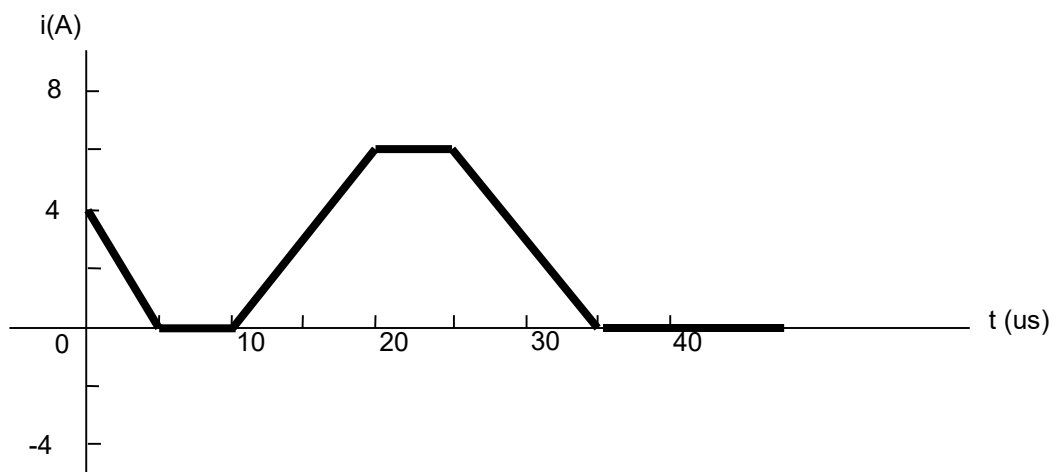
$$V(t) = 2 \cdot 10^5 \int_{35 \cdot 10^{-6}}^t 0dt + V(35 \cdot 10^{-6}) = 24V$$

f)



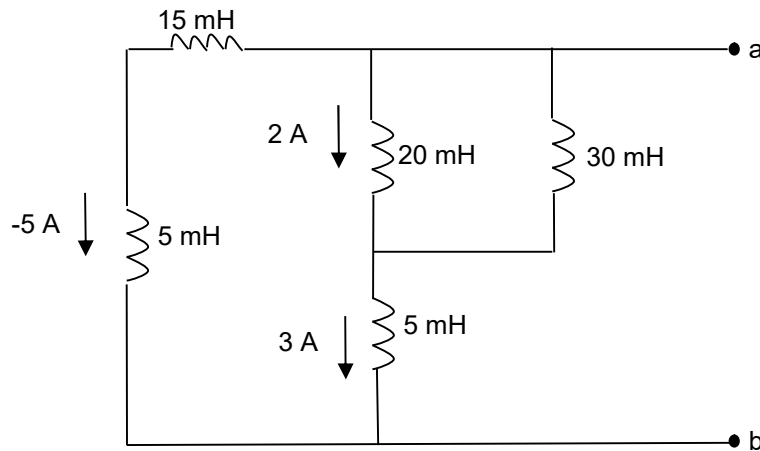
3U. The current signal is shown in the following figure is applied to a 5  $\mu\text{F}$  capacitor. The initial voltage on the capacitor is 12 V drop in the reference direction of the current. Assume the passive sign convention. Derive the expression for the capacitor voltage for the time intervals in (a)-(e).

- $0 \leq t \leq 5 \text{ us}$
- $5 \text{ us} \leq t \leq 20 \text{ us}$
- $20 \text{ us} \leq t \leq 25 \text{ us}$
- $25 \text{ us} \leq t \leq 35 \text{ us}$
- $35 \text{ us} \leq t \leq \infty$
- Sketch  $v(t)$  over the interval a)  $0 \text{ us} \leq t \leq 300 \text{ us}$ .



**Solution:**

4S. For the following circuit, find the equivalent Inductor (fully define  $L_{\text{eq}}$ ) with respect to terminals a and b:

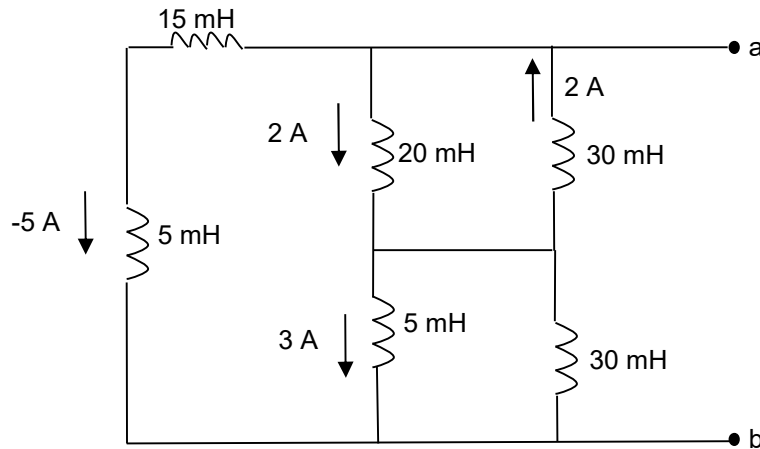


**Solution**

$I = -5 + 3 = -2 \text{ A}$  "Initial Condition"

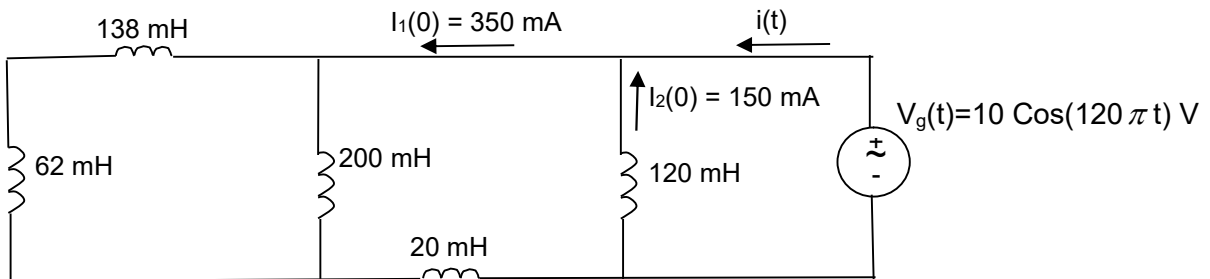
$L_{eq} = (5 + 15) \parallel (20 \parallel 30 + 5) = 9.19 \text{ mH}$

4U. For the following circuit, find the equivalent Inductor (fully define  $L_{eq}$ ) with respect to terminals a and b:



**Solution:**

5S. In the following circuit, Write  $i(t)$  equation for  $t \geq 0$  sec.



**Solution:**

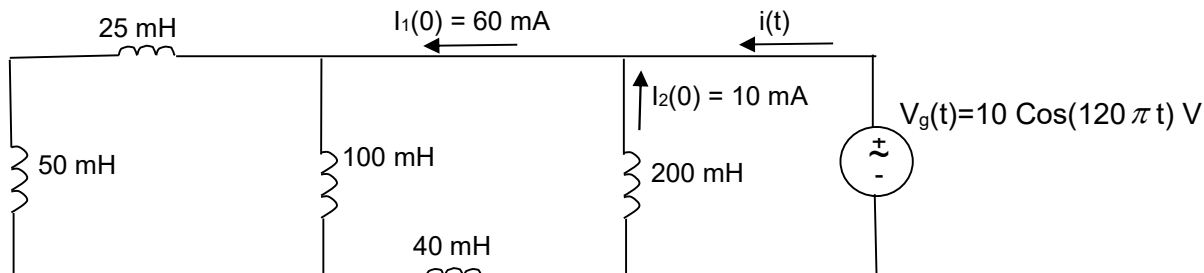
$L_{eq} = ((138+62) \parallel 200) + 20 \parallel (120) = 60 \text{ mH}$

$$i_{eq}(2) = 350 - 150 = 200 \text{ mA}$$

$$i(t) = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0) = \frac{1}{0.06} \int_0^t 10 \cos(120\pi\tau) d\tau + i(0) = + \frac{10}{0.06 \times 120\pi} \sin(120\pi\tau) \Big|_0^t + 0.2$$

$$i(t) = 0.44 \sin(120\pi t) + 0.2 \text{ A}$$

5U. In the following circuit, Write  $i(t)$  equation for  $t \geq 0$  sec.

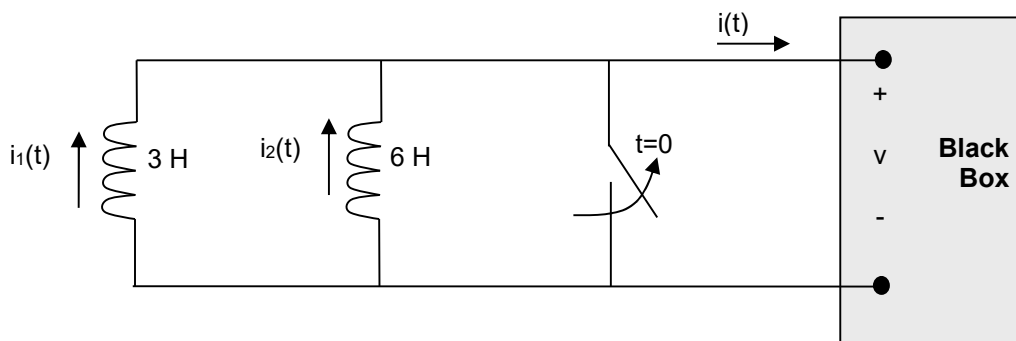


**Solution:**

6S. The two parallel inductors in the following figure are connected across the terminals of a black box at  $t=0$ .

The resulting voltage  $v$  for  $t \geq 0$  is known to be  $12e^{-t}$  V. It is also known that  $i_1(0) = 2$  A and  $i_2(0) = 4$  A.

- Replace the original inductors with an equivalent inductor and find  $i(t)$  for  $t \geq 0$ .
- Find  $i_1(t)$  for  $t \geq 0$ .
- Find  $i_2(t)$  for  $t \geq 0$ .
- How much energy is delivered to the black box in the time interval  $0 \leq t \leq \infty$ ?
- How much energy was initially stored in the parallel inductors?
- How much energy is trapped in the ideal inductors?
- Do your solutions for  $i_1$  and  $i_2$  agree with the answer obtained in (f)?



**Solution:**

$$V(t) = 12e^{-t} \text{ V for } t \geq 0$$

$$i_1(0) = 2 \text{ A}$$

$$i_2(0) = 4 \text{ A}$$

- $L_{eq} = (3 \parallel 6) = 1 / [(1/3) + (1/6)] = 2 \text{ H}$   
 $i(0) = i_1(0) + i_2(0) = 2 + 4 = 6$  [NOTE: this is entering the negative terminal of Inductors]

$$i(t) = \frac{-1}{L} \int_0^t V dt + i(0) = \frac{-1}{2} \int_0^t 12e^{-t} dt + 6 = 6e^{-t} \Big|_0^t + 6 = 6e^{-t} \text{ A}$$

- \*\*Refer to passive sign convention. Here  $i_1(t)$  is entering Negative Voltage Point

$$i_1(t) = -\frac{1}{L_1} \int_0^t V dt + i_1(0) = -\frac{1}{3} \int_0^t 12e^{-t} dt + 2$$

$$= (+4)[e^{-t}]_0^t + 2 = 4e^{-t} - 2A$$

$$c) i_2(t) = -\frac{1}{L_2} \int_0^t V dt + i_2(0) = -\frac{1}{6} \int_0^t 12e^{-t} + 4 = +2[e^{-t}]_0^t + 4 = 2e^{-t} + 2 A$$

d) How much Energy is delivered to Black Box?

$$W = (1/2)L_{eq}i^2 = (1/2)(2)(6e^{-t})^2 = 36e^{-2t} \text{ Joules for } t \geq 0$$

$$t = 0 \rightarrow W_0 = 36 \text{ Joules}$$

$$t = \infty \rightarrow W_\infty = 0 \text{ Joules} \rightarrow W_{\text{Black Box}} (0 \leq t \leq \infty) = 36 \text{ joules}$$

e) Initial Energy (t=0)

$$L_1 \rightarrow W_0 = (1/2)L_1 i_1^2 = (1/2)(3)(4e^{-t}-2)^2 = (1/2)(3)(4-2)^2$$

$$W_{0L1} = 6 \text{ Joules}$$

$$L_2 \rightarrow W_0 = (1/2)L_2 i_2^2 = (1/2)(6)(2e^{-t}+2)^2 = \frac{1}{2}(6)(2+2)^2$$

$$W_{0L2} = 48 \text{ Joules}$$

$$W_{\text{total stored energy}} = W_{0L1} + W_{0L2} = 6 + 48 = 54 \text{ Joules}$$

f) Trapped energy in Inductors? (t = ∞)

$$L_1 \rightarrow W_{L1} = (1/2)L_1 i_1^2 = (1/2)(3)(4e^{-t}-2)^2 = (1/2)(3)(4e^{-\infty}-2)^2 = 6 \text{ Joules}$$

$$L_2 \rightarrow W_{L2} = (1/2)L_2 i_2^2 = (1/2)(6)(2e^{-t}+2)^2 = (1/2)(6)(4e^{-\infty}-2)^2 = 12 \text{ Joules}$$

$$W_{\text{total trapped}} = 6 + 12 = 18 \text{ Joules}$$

g) Energy delivered to Black Box = Inductor Initial Energy – Inductor Trapped Energy

$$36 = 54 - 18$$

$$36 = 36$$

Yes, they are equal!

6U. The two parallel inductors in the following figure are connected across the terminals of a black box at t=0.

The resulting voltage v for t ≥ 0 is known to be 8e<sup>-2t</sup> V. It is also known that i<sub>1</sub>(0) = 5 A and i<sub>2</sub>(0) = 8 A.

a) Replace the original inductors with an equivalent inductor and find i(t) for t ≥ 0.

b) Find i<sub>1</sub>(t) for t ≥ 0.

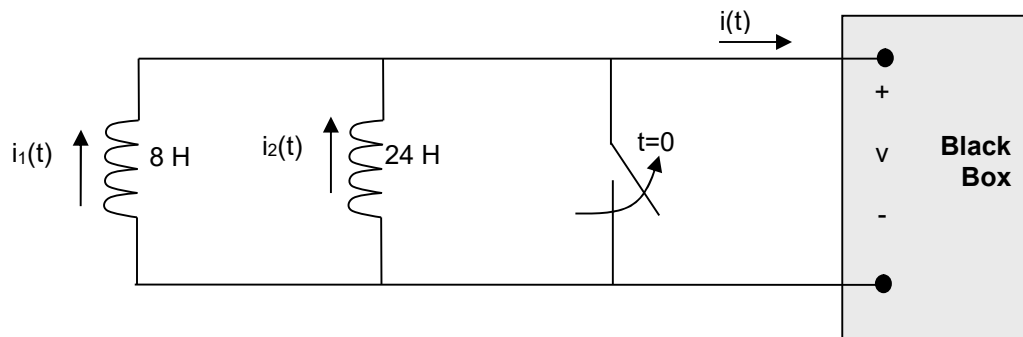
c) Find i<sub>2</sub>(t) for t ≥ 0.

d) How much energy is delivered to the black box in the time interval 0 ≤ t ≤ ∞?

e) How much energy was initially stored in the parallel inductors?

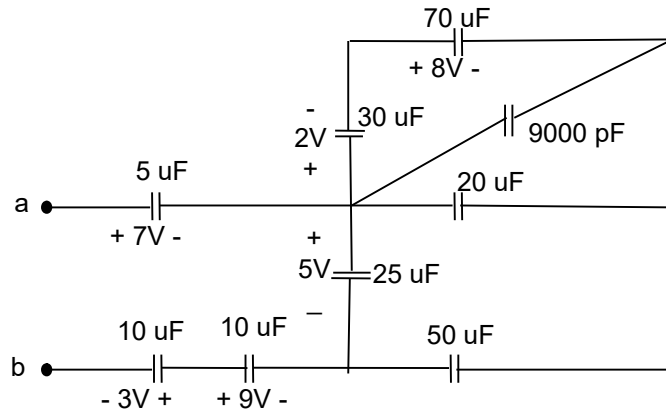
f) How much energy is trapped in the ideal inductors?

g) Do your solutions for i<sub>1</sub> and i<sub>2</sub> agree with the answer obtained in (f)?



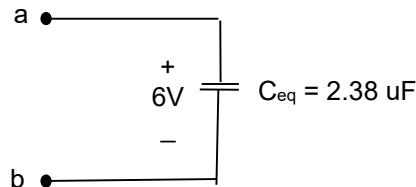
**Solution:**

7S. Calculate the equivalent capacitance with respect to terminals a and b of the following circuit:

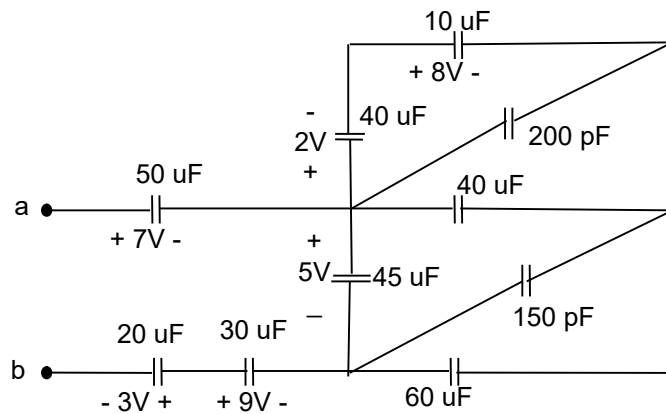


**Solution:**

Series: 30 & 70 uF  $\rightarrow 1/(1/30 + 1/70) = 21$  uF with Initial cond. 2+8 = 10 V  
 Parallel: 21, 20, .009 uf  $\rightarrow 21+20+0.009= 41$  uF with Initial cond. 10 V  
 Series: 50, 41 uf  $\rightarrow 1/(1/50 + 1/41) = 22.5$  uF  
 Parallel: 22.5, 25 uF  $\rightarrow 25+22.5 = 47.5$  uF with Initial cond. 5 V  
 Series: 5, 47.5, 10, 10 uF  $\rightarrow 1/(1/5 + 1/47.5+1/10+1/10) = 2.38$  uF  
 with initial Cond. +7 + 5 - 9 + 3 = 6 V



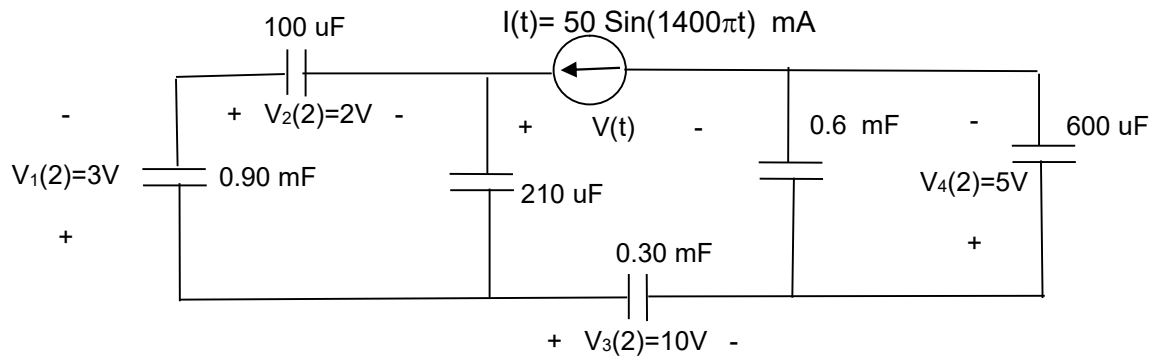
7U. Calculate the equivalent capacitance with respect to terminals a and b of the following circuit:



**Solution:**

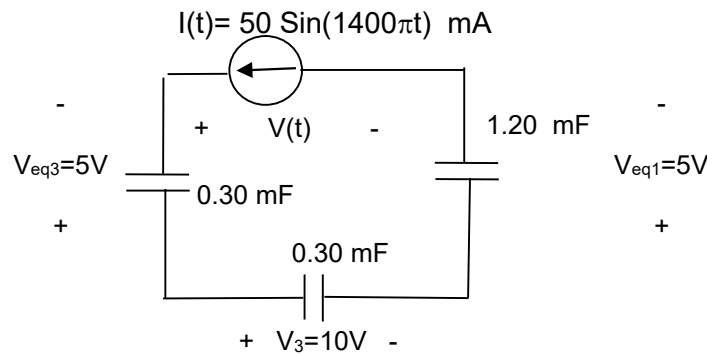


8S. In the following circuit, find the equation for voltage across the current source,  $v(t)$  for  $t \geq 2$  sec.

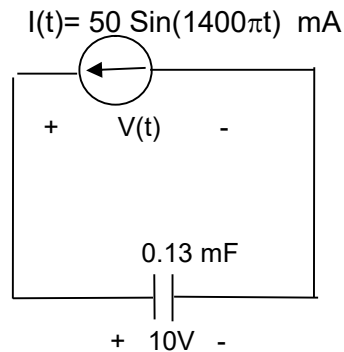


**Solution:**

0.6 mF in Parallel with 600  $\mu$ F  $\rightarrow$   $C_{eq1} = 1.20$  mF with  $V = 5$  V  
 0.9 mF in Series with 0.1 mF  $\rightarrow$   $C_{eq2} = 0.09$  mF with  $V = 5$  V  
 0.09 mF in Parallel with 0.21 mF  $\rightarrow$   $C_{eq3} = 0.30$  mF with  $V = 5$  V



0.30 mF, 0.30 mF and 1.20 mF are in series  $\rightarrow$   $C_{eq4} = 0.13$  mF with  $V = 10 - 5 + 5 = 10$  V

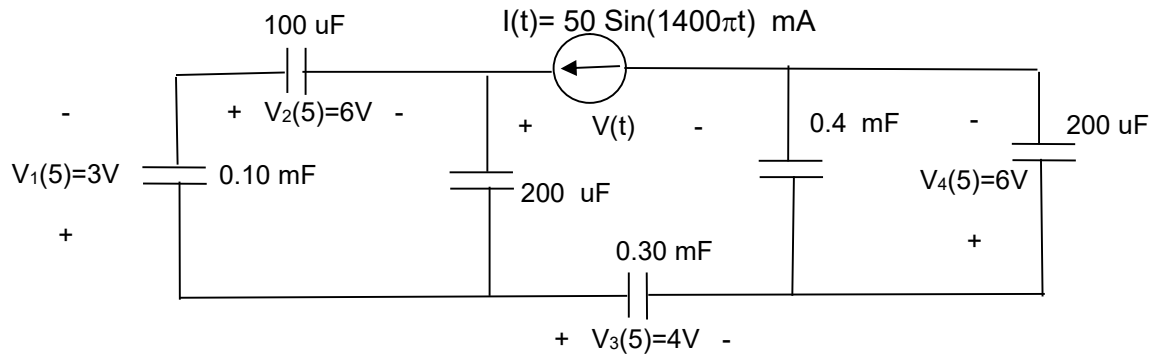


$$v(t) = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0) = \frac{1}{0.133 \cdot 10^{-3}} \int_2^t 50 \cdot 10^{-3} \cdot \sin(1400\pi\tau) d\tau + v(2)$$

$$v(t) = -\frac{50 \cdot 10^{-3}}{.133 \cdot 10^{-3} \cdot 1400\pi} \cos(1400\pi t) \Big|_2^t + 10$$

$$v(t) = -0.085 \cos(1400\pi t) + 10.085 \text{ V}$$

8U. In the following circuit, find the equation for voltage across the current source,  $v(t)$  for  $t \geq 5$  sec.

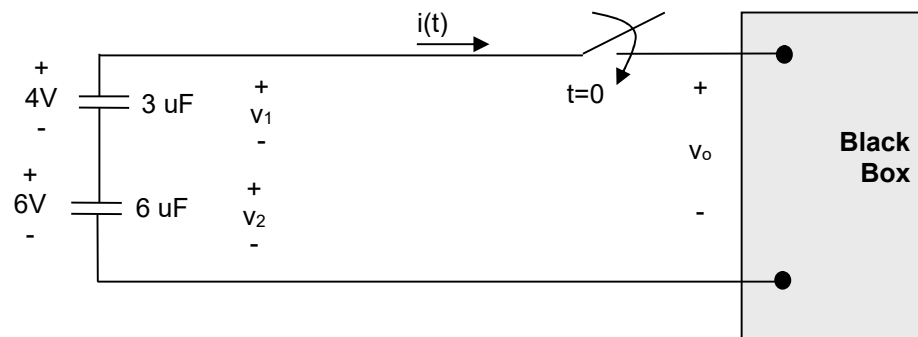


**Solution:**

8Sb. The two series-connected capacitors are connected to the terminals of a black box at  $t=0$  as shown below.

The resulting current  $i(t)$  for  $t \geq 0$  is known to be  $20e^{-t}$  uA.

- Replace the original capacitors with an equivalent capacitor and find  $v_o(t)$  for  $t \geq 0$ .
- Find  $v_1(t)$  for  $t \geq 0$ .
- Find  $v_2(t)$  for  $t \geq 0$ .
- How much energy is delivered to the black box in the time interval  $0 \leq t \leq \infty$ ?
- How much energy was initially stored in the series capacitors?
- How much energy is trapped in the ideal capacitors?
- Do the solutions for  $v_1$  and  $v_2$  agree with the answer obtained in (f)?



**Solution:**

- $i(t) = 20e^{-t}$  uA  $t \geq 0$   
find  $C_{eq}$  and  $V_0(t)$   
 $(1/C_{eq}) = (1/C_1) + (1/C_2) \rightarrow C_{eq} = 2$  uF

Note again that the current is flowing from negative terminal to positive which means need to have “-“ between  $I$  and  $V$  relation

$$V_0 = \frac{-1}{Ceq} \int_0^t i dt + V_0(0)$$

$$V_0 = \frac{-10^6}{2} \int_0^t 20e^{-t} * 10^{-6} dt + 10$$

$$V_0 = 10e^{-t} \Big|_0^t + 10 = 10e^{-t}$$

b)

$$V_1(t) = -\frac{1}{3 * 10^{-6}} \int_0^t 20e^{-t} * 10^{-6} dt + 4 = -\frac{20}{3} [-e^{-t}]_0^t + 4 = 6.67e^{-t} - 2.67 V$$

c)

$$V_2(t) = -\frac{1}{6 * 10^{-6}} \int_0^t 20e^{-t} * 10^{-6} dt + 6 = -\frac{20}{6} [-e^{-t}] + 6 = 3.33e^{-t} + 2.67 V$$

d) We know from step a that

$$V_0 = \frac{-1}{Ceq} \int_0^t i dt + V_0(0)$$

$$V_0 = \frac{-10^6}{2} \int_0^t 20e^{-t} * 10^{-6} dt + 10$$

$$V_0 = 10e^{-t} \Big|_0^t + 10 = 10e^{-t}$$

$$W = (1/2) \int_0^{\infty} 10e^{-t} * 20e^{-t} * 10^{-6} = 10^{-6} (100) \int_0^{\infty} e^{-2t} = 100 * 10^{-6}$$

$$W = 100 \mu\text{Joules}$$

e) @ t=0

$$W_{1\Box} = (1/2)cv_1(t)^2 = (1/2)(3*10^{-6})(4)^2 = 24 \mu\text{Joules}$$

$$W_2 = (1/2)cv_2(t)^2 = 2 = (1/2)(6*10^{-6})(6)^2 = 108 \mu\text{Joules}$$

$$24 \mu\text{Joules} + 108 \mu\text{Joules} = 132 \mu\text{Joules}$$

f) @ t=∞

$$W_1 = (1/2)CV_1(t=\infty)^2 = (1/2)(3*10^{-6})(-2.67)^2 = 10.69 \mu\text{Joules}$$

$$W_2 = (1/2)CV_2(t=\infty)^2 = (1/2)(6*10^{-6})(2.67)^2 = 21.38 \mu\text{Joules}$$

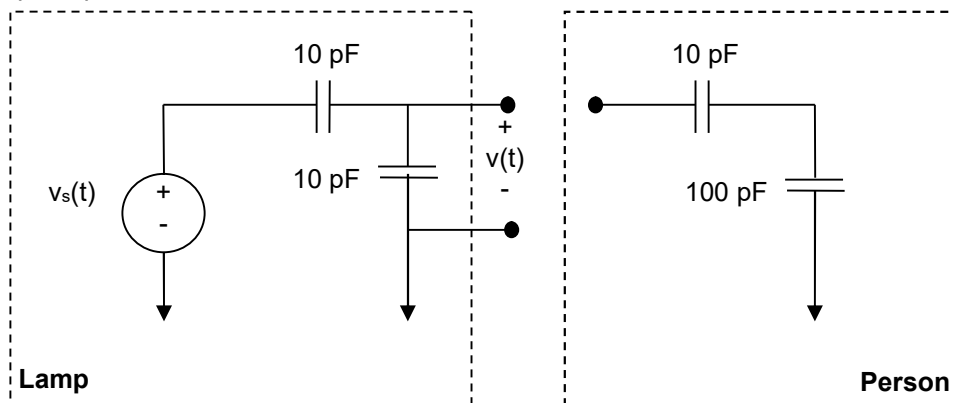
$$10.69 \mu\text{Joules} + 21.31 \mu\text{Joules} = 32.0 \mu\text{Joules}$$

g) 100 μJoules + 32 μJoules = 132 μJoules (Verified)

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8Sc. Some lamps are made to turn on or off when the base is touched. These use a one-terminal variation of the capacitive switch design. Calculate the change in voltage v(t) when a person touches the lamp. Assume that all capacitors are initially discharged and a person can be modeled with a series of 10 pF and

100 pF capacitors.



**Solution:**

Note that capacitors divide voltage as shown below:

$V_s(t) = \frac{1}{C_1} \int i(t) dt + \frac{1}{C_2} \int i(t) dt$ $V_s(t) = \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \int i(t) dt$ $\int i(t) dt = \frac{C_1 C_2}{C_1 + C_2} V_s(t)$ $V(t) = \frac{1}{C_2} \int i(t) dt = \frac{C_1}{C_1 + C_2} V_s(t)$	
--	--

Using the above analysis result for the circuit when no person touching, only 10 pF goes to ground. Therefore

$$v(t) = (10/20) v_s(t) = 0.5 v_s(t)$$

With figure touching, then the capacitors representing person must be considered. There we have to calculate the equivalent capacitor to ground:

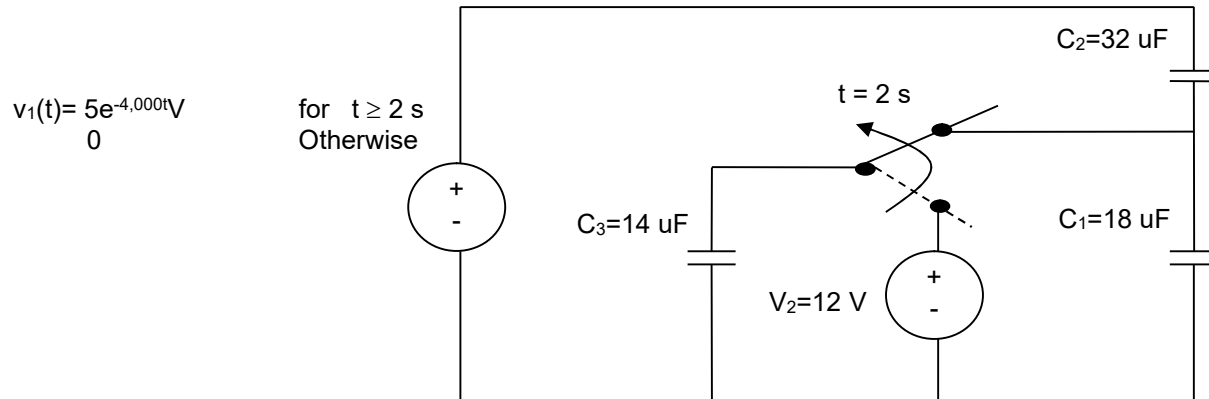
$$C_{eq} = \frac{1}{\frac{1}{100} + \frac{1}{10}} + 10 = 19.091 \text{ pF}$$

$$v(t) = \frac{10}{19.091 + 10} v_s(t) = 0.344 v_s(t)$$

Therefore

$$\Delta v(t) = 0.156 v_s(t)$$

9S. For the following circuit find the current through  $C_2$  for  $t \geq 2$  s, and the total energy delivered to  $C_2$ .



**Solution:**

$$V_{C3}(2) = 12 \text{ V}, \quad V_{C2}(2) = 0 \text{ V}, \quad V_{C1}(2) = 0 \text{ V}$$

$$t \geq 2 \text{ s}$$

$$C_{eq} = (18 + 14) \parallel 32 = 16 \text{ uF}$$

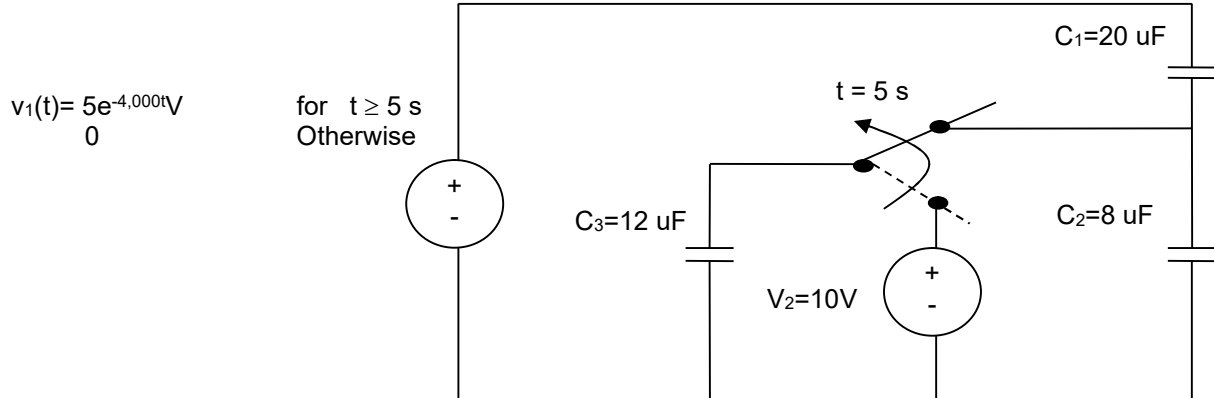
$$i(t) = Cdv/dt = 16 \times 10^{-6} (-20,000 e^{-4,000t}) = -0.32 e^{-4,000t} \text{ A}$$

$$v_{c2} = \frac{1}{C_2} \int_2^t i dt = \frac{1}{32 \times 10^{-6}} \int_2^t \{-0.32 e^{-4000t}\} dt + v_{c2}(2) = 2.5 \{e^{-4000t} - e^{-8000}\} + 0$$

$$v_{c2} = 2.5e^{-4000t} - 2.5e^{-8000} \text{ V}$$

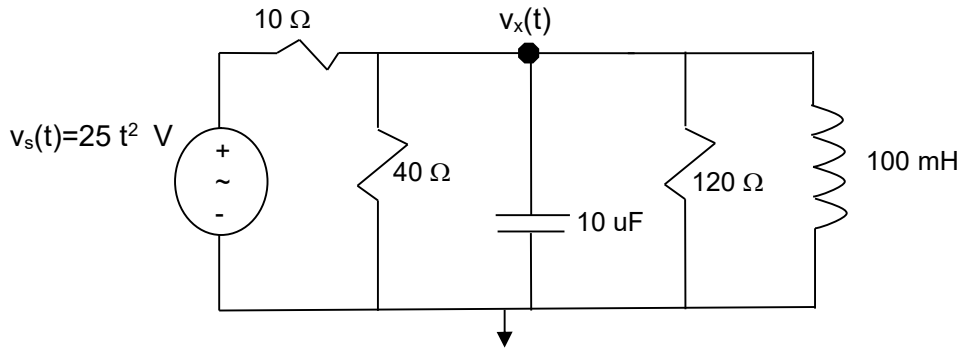
$$E_{c2,t \rightarrow \infty} = \frac{1}{2} CV^2 = \frac{32 \times 10^{-6}}{2} [2.5e^{-4000t} - 2.5e^{-8000}]^2 \text{ Joules}$$

9U. For the following circuit find the current through  $C_1$  for  $t \geq 5$  s, and the total energy delivered to  $C_3$ .



**Solution:**

10S. Write an differential equation in term of  $v_x(t)$  for the following circuit:



**Solution:**

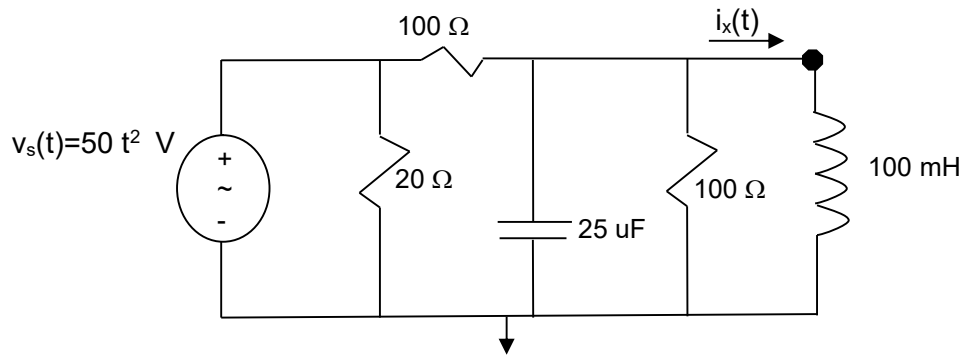
Use KCL at Node  $v_x(t) \rightarrow \frac{v_x - 25t^2}{10} + \frac{v_x}{40} + 10^{-5} \frac{dv_x}{dt} + \frac{v_x}{120} + \frac{1}{0.1} \int_{t_0}^t v_x(\tau) d\tau + i_L(t_0) = 0$

$$0.1 \frac{dv_x}{dt} - 5t + 0.025 \frac{dv_x}{dt} + 10^{-5} \frac{d^2 v_x}{dt^2} + \frac{1}{120} \frac{dv_x}{dt} + \frac{v_x}{0.1} = 0$$

Take a derivate of both side  $\rightarrow$

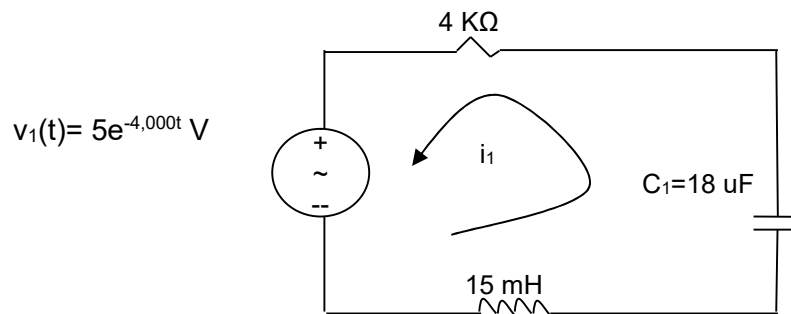
$$\frac{d^2 v_x}{dt^2} + 13,333.33 \frac{dv_x}{dt} + 10^6 v_x - 5 \times 10^5 t = 0$$

10U. Write an differential equation in term of  $i_x(t)$  for the following circuit:



**Solution:**

10Sb. Use KVL to write a differential equation in terms of  $i_1$  for the following circuit:



**Solution**

$$4000i_1 + 5e^{-4000t} + 15 \cdot 10^{-3} \frac{di_1}{dt} + \frac{10^6}{18} \int i_1 dt = 0$$

derivative :

$$15 \cdot 10^{-3} \frac{d^2 i_1}{dt^2} + 4000 \frac{di_1}{dt} + \frac{10^6}{18} i_1 = 20,000e^{-4000t}$$