

## Fundamentals of Electrical Circuits - Chapter 9

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- 1S. Consider the Sinusoidal Voltage  $\rightarrow v(t) = 40 \cos(100\pi t + 60^\circ)$  V
- What is the Maximum Amplitude of the Voltage?
  - What is the frequency in Hertz?
  - What is the frequency in Radians per second?
  - What is the phase angle in radians?
  - What is the phase angle in degrees?
  - What is the period in milliseconds?
  - What is the first time after  $t=0$  that  $v=-40$  V?
  - The sinusoidal function is shifted  $10/3$  ms to the right along the time axis. What is the expression for  $v(t)$ ?
  - What is the minimum number of milliseconds that the original  $v(t)$  function must be shifted to the right so that it becomes equal to  $40 \sin(100\pi t)$ ?
  - What is the minimum number of milliseconds that the original  $v(t)$  function must be shifted to the left so that it becomes equal to  $40 \cos(100\pi t)$ ?

**Solution:**

a)  $V_{\max} = 40$  V

b)  $\omega = 2\pi f = 100\pi \rightarrow f = (100\pi)/2\pi = 50$  Hz

c)  $\omega = 100\pi$  rad./Sec.

d)  $\Phi = (60^\circ)(\pi \text{ rad}/180^\circ) = \pi/3$

e)  $\Phi = (60^\circ)$

f)  $T = 1/f = 1/50 = 20$  ms

g)  $t = ?$  where  $v(t) = -40 = 40\cos(100\pi t + 60^\circ)$   
 $\cos(100\pi t + 60^\circ) = -1 = \cos((2k+1)\pi)$  where  $k = 0, 1, 2, 3, \dots$   
 $100\pi t + \pi/3 = (2k+1)\pi \rightarrow t = (2k + 1 - 1/3)/100 = (6k + 2)/300$   
first time after  $t=0$ ,  $v(t) = -40$  at  $t = 2/300$  ( $K=0$ )

h)  $\Delta t = 10/3$  ms "shift to the right on time axis"  
 $\Delta\Phi = \Delta t * \omega = (1/300 \text{ s})(100\pi \text{ rad/s}) = \pi/3$   
 $v_{\text{shifted}}(t) = 40\cos(100\pi t + \pi/3 - \pi/3) = 40\cos(100\pi t)$

i) We need  $\Delta\Phi = -\pi/2 - \pi/3 = -5\pi/6$  (to convert sine to cosine and then shift to zero)  
Now convert to time from radian  
 $\Delta t = \Delta\Phi / \omega = (-5\pi/6) / (100\pi) = -5/600 = -8.33$  ms (negative indicates shift to the right)

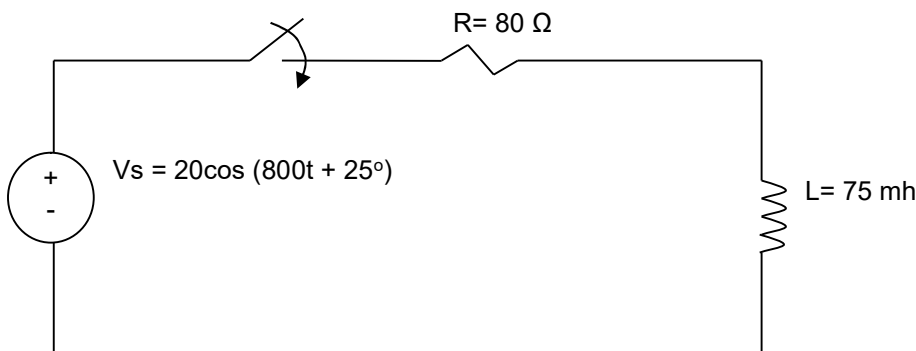
j) We need go forward by  $2\pi$  less  $\pi/3$  or  $60^\circ$  in order to get the desired sign  $40 \cos(100\pi t)$   
 $\Delta\Phi = 2\pi - \pi/3 = 5\pi/3$   
Now convert radian to time  
 $\Delta t = \Delta\Phi / \omega = (5\pi/3) / (100\pi) = 16.7$  ms (positive indicates shift to the left)

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- 1U. Consider the Sinusoidal Voltage  $\rightarrow v(t) = 15 \cos(250\pi t + 75^\circ)$  V
- What is the Maximum Amplitude of the Voltage?
  - What is the frequency in Hertz?
  - What is the frequency in Radians per second?
  - What is the phase angle in radians?
  - What is the phase angle in degrees?
  - What is the period in milliseconds?
  - What is the first time after  $t=0$  that  $v=-10$  V?

- h) The sinusoidal function is shifted 10/3 ms to the right along the time axis. What is the expression for  $v(t)$ ?
- i) What is the minimum number of milliseconds that the function must be shifted to the right if the expression for  $v(t)$  is  $15 \sin(250\pi t)$
- j) What is the minimum number of milliseconds that the function must be shifted to the left if the expression for  $v(t)$  is  $15\cos(250\pi t)$  V.

**Solution:**

- 2S. The Voltage applied to the circuit shown below at  $t=0$  is  $20\cos(800t + 25^\circ)$ . The circuit resistance is  $80 \Omega$ , and the initial current in the 75 mh inductor is zero.
- Find  $i(t)$  for  $t \geq 0$ .
  - Write the expression for the transient and steady state component of  $i(t)$ .
  - Find the numerical value of  $I$  after the switch has been closed for 1.875 ms.
  - What are the maximum amplitude, frequency (rad./s), and phase angle of the steady state current?
  - By how many degrees are the voltage and steady state current out of phase?



**Solution:**

- a) Since Here we are asked to find the total response  $i(t)$ , we can use the following equation where the first term is the transient component and second term ins the steady state component.

$$i(t) = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta) \quad \text{where } \theta = \tan^{-1}(\omega L / R)$$

$$\omega = 800, \quad \phi = 25^\circ, \quad V_m = 20 \quad V$$

$$\theta = \tan^{-1}(800 * 0.075 / 80) = 36.87^\circ$$

$$i(t) = \frac{-20}{\sqrt{80^2 + 800^2 (.075)^2}} \cos(25^\circ - 36.87^\circ) e^{-(80/0.075)t} + \frac{20}{\sqrt{80^2 + 800^2 (.075)^2}} \cos(800t + 25^\circ - 36.87^\circ)$$

$$i(t) = -0.2 \cos(-11.87^\circ) e^{-1066.67t} + 0.2 \cos(800t - 11.87^\circ)$$

- b)

$$\text{Transient part of } i(t) = i(t) = -0.2 \cos(-11.87^\circ) e^{-1066.67t}$$

$$\text{Steady State part of } i(t) = +0.2 \cos(800t - 11.87^\circ)$$

- c) Numerial value of  $I(t=1.875 \text{ ms})$

$$i(1.875 \text{ ms}) = -0.2 \cos(-11.87^\circ * \pi / 180) e^{-(1066.67 * 1.875 / 1000)} + 0.2 \cos(800 * 1.87 / 1000 - 11.87^\circ * \pi / 180)$$

Note all angles are converted to radian first.

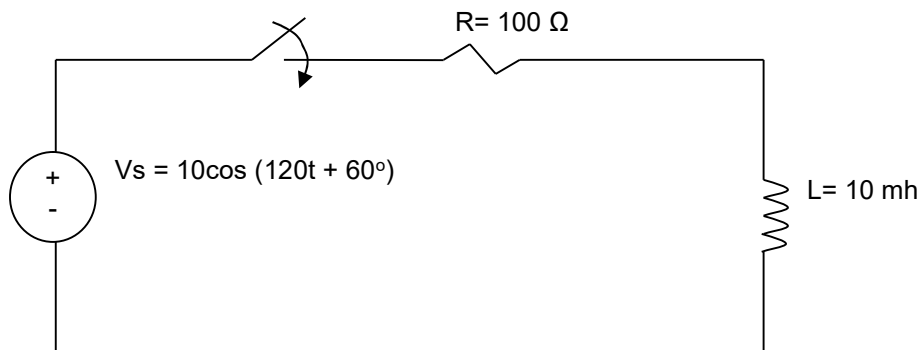
$$i(1.875 \text{ ms}) = 29 \text{ mA}$$

- d) For steady state part:  
Maximum Amplitude =  $I_{\text{max}} = 0.2$   
Angular Frequency (Rad/s) =  $\omega = 800 \text{ rad/sec.}$   
Phase Shift =  $\Phi = -11.87^\circ$
- e) Phase difference =  $25 - (-11.87) = 36.87^\circ$

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2U. The Voltage applied to the circuit shown below at  $t=0$  is  $10\cos(120t + 60^\circ)$ . The circuit resistance is  $100 \Omega$ , and the initial current in the  $10 \text{ mh}$  inductor is zero.

- a) Find  $i(t)$  for  $t \geq 0$ .  
b) Write the expression for the transient and steady state component of  $i(t)$ .  
c) Find the numerical value of  $I$  after the switch has been closed for  $2 \text{ ms}$ .  
d) What are the maximum amplitude, frequency (rad./s), and phase angle of the steady state current?  
e) By how many degrees are the voltage and steady state current out of phase?

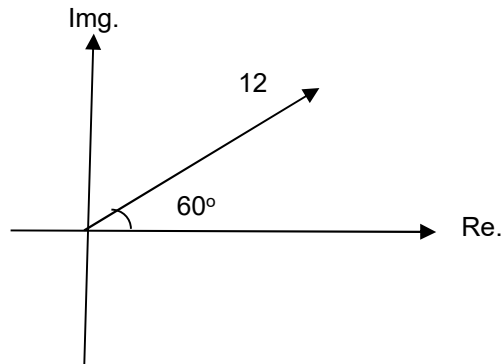


**Solution:**

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3S. Write the time domain equation and phasor Polar, Rectangular and Angular forms for the following Signals (use  $\omega=3000 \text{ rad/sec.}$  if not provided)

a)



b)  $X = 20 e^{j\pi/3}$

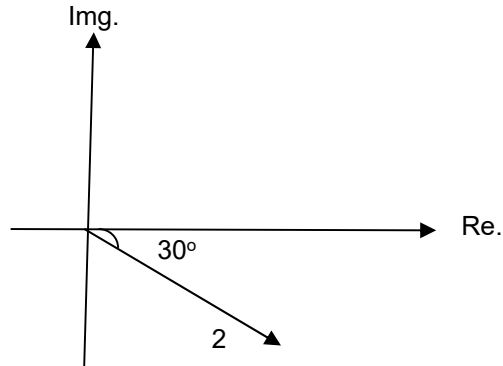
c)  $x(t) = 25 \text{ Cos}(300\pi t + \pi/3)$

**Solution:**

Part	Time Domain	Angular	Polar	Rectangular
a	$12\cos(3000t + \pi/3)$	$12 \angle \pi/3$	$12e^{j\pi/3}$	$6 + j10.4$
b	$20\cos(3000t + \pi/3)$	$20 \angle \pi/3$	$20e^{j\pi/3}$	$6 + j17.32$
c	$25 \cos(300\pi t + \pi/3)$	$25 \angle \pi/3$	$25e^{j\pi/3}$	$12.5 + j21.65$

3U. Write the time domain equation and phasor Polar, Rectangular and Angular forms for the following Signals (use  $\omega=3000$  rad/sec. if not provided)

a)



b)  $X = 25 e^{j\pi/4}$

c)  $X = 4 e^{-j\pi/3}$

d)  $x(t) = 25 \cos(300\pi t + \pi/3)$

e)  $x(t) = 10 \sin(500\pi t + \pi/2)$

f)  $X = 3 \angle 2\pi/3$

g)  $X = 25 + j32$

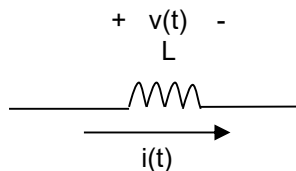
h)  $x(t) = 20 \cos(30\pi t + 6\pi/3)$

**Solution:**

4S. A 1000 Hz Sinusoidal voltage with a maximum amplitude of 200 V at  $t=0$  is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 25 A.

- What is the frequency of the inductor current?
- What is the phase angle of the voltage?
- What is the phase angle of the current?
- What is the inductive reactance of the inductor?
- What is the inductance of the inductor in millihenrys?
- What is the impedance of the inductor?

**Solution:**



- Frequency =  $f = 1000$  Hz (same as voltage)
- Since the maximum amplitude of voltage occurs at  $t=0$  then phase of voltage is 0  
 $\phi_v = 0$

- c) We know the Inductor shift current by  $-\pi/2$  rad compared to voltage  $\rightarrow$   
Current Phase Shift =  $\Phi_i = 0 - \pi/2 = -\pi/2$
- d) Inductive reactance =  $\omega L = ?$   
 $Z = V/I \rightarrow \omega L = V_{\max}/I_{\max} = 200/25 = 8 \Omega$  (reactance)
- e)  $\omega = 2\pi f = 2000\pi$   
 $\omega L = (2000\pi) L = 8 \rightarrow L = 1.27 \text{ mH}$
- f) Impedance of inductor =  $Z = j\omega L = j8 \Omega$

4U. A 2500 Hz Sinusoidal voltage with a maximum amplitude of 80 V at  $t=0$  is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 10 A.

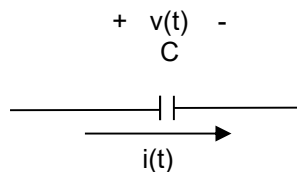
- a) What is the frequency of the inductor current?  
b) What is the phase angle of the voltage?  
c) What is the phase angle of the current?  
d) What is the inductive reactance of the inductor?  
e) What is the inductance of the inductor in millihenrys?  
f) What is the impedance of the inductor?

**Solution:**

5S. A 50 kHz sinusoidal voltage has zero phase angle and a maximum amplitude of 10 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 628.32  $\mu\text{A}$ .

- a) What is the frequency of the current in radians per second?  
b) What is the phase angle of the current?  
c) What is the capacitive reactance of the capacitor?  
d) What is the capacitance of the capacitor in microfarads?  
e) What is the impedance of the capacitor?

**Solution:**



- a) Angular Frequency (rad/s) =  $\omega = 2\pi f = 2\pi(50,000) = 100,000\pi$  rad/sec
- b) Phase Angle =  $\Phi_i = \pi/2$  Since Current is  $+\pi/2$  shift from voltage (current leads the voltage by  $\pi/2$ )
- c) Capacitive reactance =  $1/\omega C = V_{\max}/I_{\max}$   
 $1/\omega C = (10 \times 10^{-3})/628.32 = 15.92 \Omega$
- d)  $-1/\omega C = -15.92 \rightarrow C = 1/(15.92 \times 100,000\pi) = 0.2 \mu\text{F}$
- e) Impedance of the capacitor =  $Z = j(-1/\omega C) = -j15.92 \Omega$

5U. A 15 kHz sinusoidal voltage has  $45^\circ$  phase angle and a maximum amplitude of 50 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 250  $\mu\text{A}$ .

- What is the frequency of the current in radians per second?
- What is the phase angle of the current?
- What is the capacitive reactance of the capacitor?
- What is the capacitance of the capacitor in microfarads?
- What is the impedance of the capacitor?

**Solution:**

6S. Convert Steady State current  $i(t) = 10 \cos(150t - \pi/3)$  to Phasor Domain and write out the equivalent current in Angular, Polar and Rectangular forms. Also draw the Phasor representation of this signal.

**Solution**

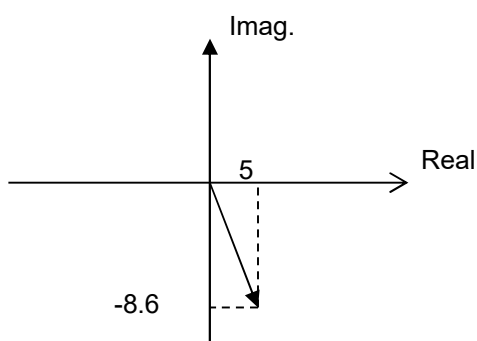
$$i(t) = 10 \cos(150t - \pi/3) = 10 \cos(150t - \pi/3) \rightarrow \omega = 150 \text{ rad/sec; Phase} = -\pi/3; V_{\max} = 10\text{V}$$

$$\text{Angular Form} \rightarrow 10 \angle -\pi/3$$

$$\text{Polar Form} \rightarrow 10 e^{-j\pi/3}$$

$$\text{Rectangular Form} \rightarrow 10\cos(-\pi/3) + j10\sin(-\pi/3) = 5 - j8.6$$

Phasor Diagram  $\rightarrow$



6U. Convert Steady State current  $i(t) = 30 \cos(200t + \pi/6)$  to Phasor Domain and write out the equivalent current in Angular, Polar and Rectangular forms. Also draw the Phasor representation of this signal.

**Solution**

6Sb. Convert Steady State voltage  $v(t) = 25 \sin(2000t + \pi/8)$  to Phasor domain and write out the equivalent voltage in Angular, Polar and Rectangular forms.

**Solution:**

$$V(t) = 25 \sin(2,000t + \pi/8) \text{ Need to convert to cosine by } (-\pi/2 \text{ shift})$$

$$V(t) = 25 \cos(2,000t + \pi/8 - \pi/2) = 25 \cos(2,000t - 3\pi/8) \rightarrow \omega = 2,000 \text{ rad/sec; Phase} = -3\pi/8; V_{\max} = 25\text{V}$$

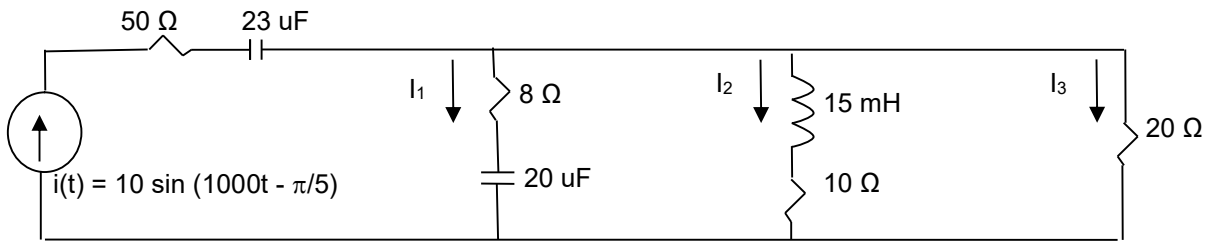
$$\text{Angular Form} \rightarrow 25 \angle -3\pi/8$$

$$\text{Polar Form} \rightarrow 25 e^{-j3\pi/8}$$

$$\text{Rectangular Form} \rightarrow 25\cos(-3\pi/8) + j25\sin(-3\pi/8) = 9.6 - j23.1$$

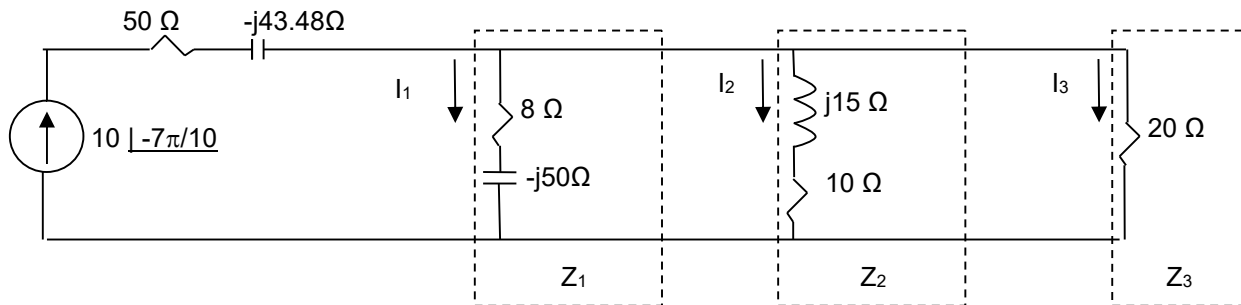
7S. Draw the phasor-domain equivalent circuit of the circuit shown below. Also identify the currents with the largest and the smallest magnitude among the three parallel branch currents ( $I_1, I_2, I_3$ )

*Hint: Finding the exact value of  $I_1, I_2$  and  $I_3$  is not required.*



**Solution**

Step 1. Convert to Phasor Domain (  $C \rightarrow 1/j\omega C$ ;  $L \rightarrow j\omega L$ ;  $R \rightarrow R$  )  
 $\omega = 1000$



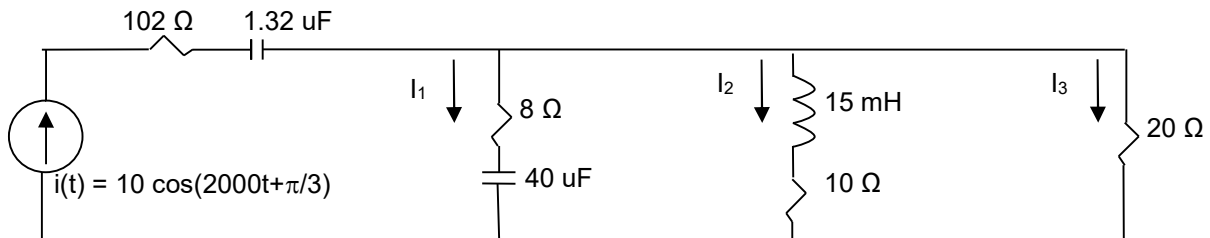
This is a current divider when  $I_n$  is inversely proportional to  $Z_n$

$|Z_1| = \sqrt{8^2 + 50^2} = 50.7\Omega$     Largest Impedance therefore Smallest current  $|I_1|$

$|Z_3| = 20\Omega$     Middle value Impedance therefore middle value current  $|I_3|$

$|Z_2| = \sqrt{10^2 + 15^2} = 18.03\Omega$     Smallest Impedance therefore Largest current  $|I_2|$

7U. Draw the Phasor domain equivalent of the circuit shown below. Also identify the currents with the largest and the smallest magnitude among the three parallel branch currents ( $I_1, I_2, I_3$ )

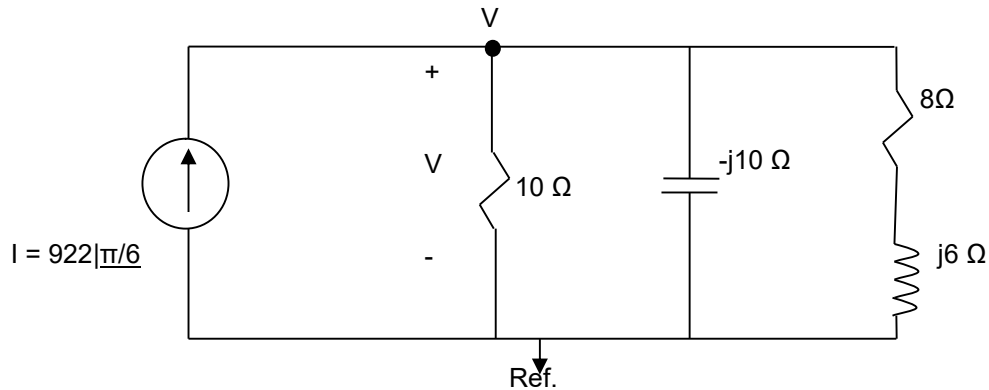


**Solution:**

- 8S. A  $10\ \Omega$  resistor and a  $5\ \mu\text{F}$  Capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of an  $8\ \Omega$  resistor and an  $300\ \mu\text{H}$  inductor. These three parallel branches are driven by a sinusoidal current source whose current is  $922\cos(20,000t + 30^\circ)\ \text{A}$ .
- Draw the Frequency-domain equivalent circuit.
  - Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.
  - Find the steady-state expression for  $v(t)$ .

**Solution:**

- Note ( $R \rightarrow R$ ,  $L \rightarrow j\omega L$ ,  $C \rightarrow 1/j\omega C = -j/\omega C$  where  $\omega = 20,000$ )



- To find  $V$ , use Node-Voltage method and write the KCL equation for node  $V$

First write the  $I$  in Rectangular form  $\rightarrow I = 922\cos(\pi/6) + j922\sin(\pi/6) = 798.48 + j461$

Now the KCL for  $V$

$$-(798.48 + j461) + V/10 + V/(-j10) + V/(8 + j6) = 0$$

$$V = (798.48 + j461) / (1/10 + j/10 + 1/(8 + j6)) = 4769.6 + j1501.2 \quad (\text{rectangular- Phasor form})$$

$$\text{Amplitude} = V_{\text{max}} = \sqrt{4769.6^2 + 1501.2^2} = 5000.2 \quad V$$

$$\text{Phase} = \Phi = \tan^{-1}\left(\frac{1501}{4769.6}\right) = 0.3 \quad \text{Rad. or } 17.5^\circ$$

$$V = 5000 \angle 17.5^\circ$$

- Steady State  $v(t) = 5000\cos(20,000t + 17.5^\circ)$

- 8U. A  $20\ \Omega$  resistor and a  $50\ \mu\text{F}$  Capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of an  $50\ \Omega$  resistor and an  $200\ \mu\text{H}$  inductor. These three parallel branches are driven by a sinusoidal current source whose current is  $120\cos(10,000t + 45^\circ)\ \text{A}$ .

- Draw the Frequency-domain equivalent circuit.
- Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.
- Find the steady-state expression for  $v(t)$ .

**Solution:**

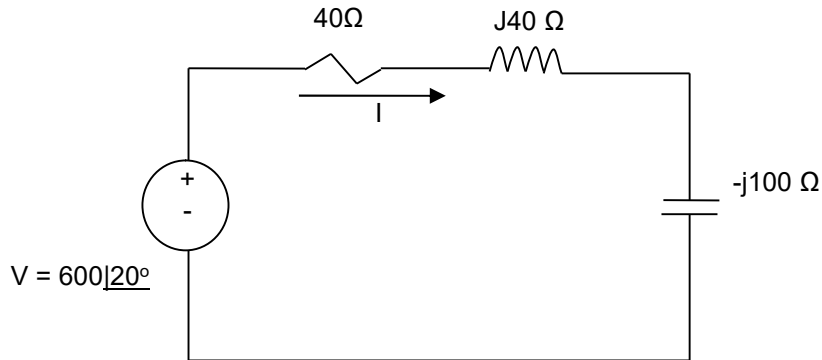
- 9S. A  $40\ \Omega$  resistor, a  $5\ \text{mH}$  inductor and a  $1.25\ \mu\text{F}$  capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is  $600\cos(8000t + 20^\circ)\ \text{V}$ .



- Draw the frequency-domain equivalent circuit.
- Reference the current in the direction of the voltage rise across the source, and find the phasor current.
- Find the steady state expression for  $i(t)$ .

**Solution:**

- Note:  $R \rightarrow R$ ,  $L \rightarrow j\omega L$ ,  $C \rightarrow 1/j\omega C = -j/\omega C$  where  $\omega = 8,000$



- $I = V/Z$   
If we write  $Z$  also in angular form then we can divide magnitudes and subtract phase to get the angular form of  $I$

$$Z = (40 + j40 - j100) = (40 - j60) = \sqrt{40^2 + 60^2} \left| \tan^{-1}\left(-\frac{60}{40}\right) \right| = 72.11 \angle -56.31^\circ$$

$$I = (600 \angle 20^\circ) / (72.11 \angle -56.31^\circ) = (600/72.11) \angle (20 - (-56.31))^\circ$$

$$I = 8.32 \angle 76.31^\circ \text{ A}$$

- Steady State  $i(t) = 8.32 \cos(8,000t + 76.31^\circ) \text{ A}$

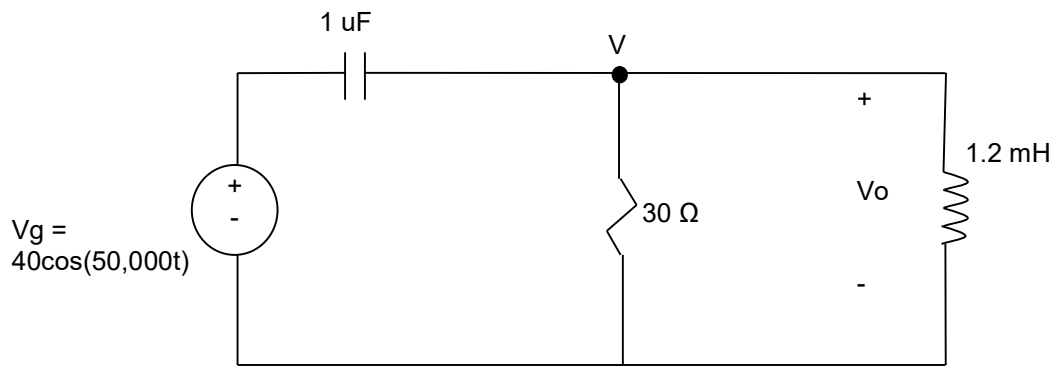
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9U. A  $40 \Omega$  resistor, a  $5 \text{ mH}$  inductor and a  $1.25 \mu\text{F}$  capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is  $600 \sin(8000t + 110^\circ) \text{ V}$ .

- Draw the frequency-domain equivalent circuit.
- Reference the current in the direction of the voltage rise across the source, and find the phasor current.
- Find the steady state expression for  $i(t)$ .

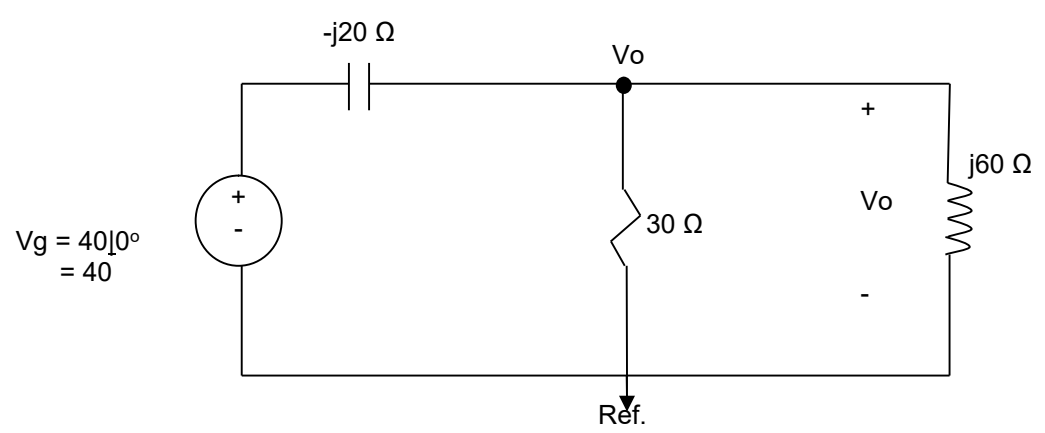
**Solution:**

10S. The Circuit in the following figure is operating in the sinusoidal steady state. Find the steady-state expression for  $v_o(t)$  if  $v_g(t) = 40 \cos(50,000t)$  V.



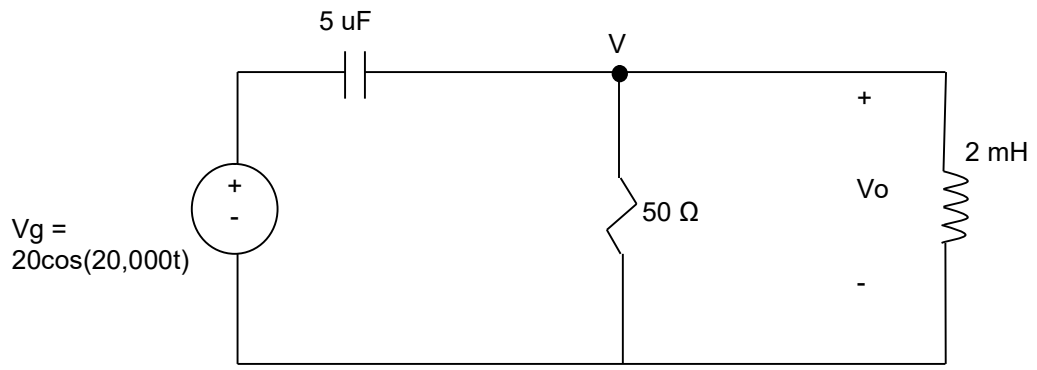
**Solution:**

Step 1) Convert the circuit to Frequency domain using phasor transformation  
 Note ( $R \rightarrow R$ ,  $L \rightarrow j\omega L$ ,  $C \rightarrow 1/j\omega C = -j/\omega C$  where  $\omega = 50,000$ )



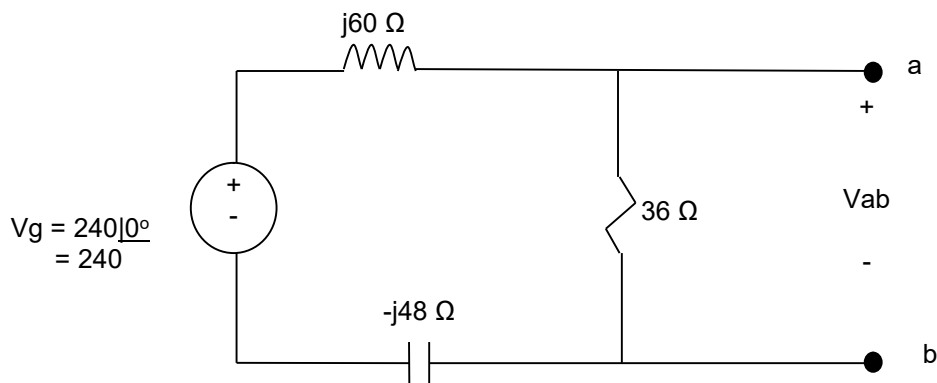
Step 2) Decide on Analysis method, Node Voltage is the simplest approach with 1 KCL equation  
 So write the KCL for Node  $V_o$   
 $(V_o - 40) / -j20 + V_o / 30 + V_o / j60 = 0$   
 $V_o = (-40/j20) / (1/-j20 + 1/30 + 1/j60) = 30 + j30 = 42.43 \angle 45^\circ$   
 Steady State expression  $\rightarrow v(t) = 42.43 \cos(50,000t + 45^\circ)$

10U. The Circuit in the following figure is operating in the sinusoidal steady state. Find the steady-state expression for  $v_o(t)$  if  $v_g(t) = 20 \cos(20,000t)$  V.



**Solution:**

- 11S. Find the Thevenin equivalent circuit with respect to the terminals a, b for the circuit shown in the following figure.



**Solution:**

Step 1) Find the  $V_{oc} = V_{th}$

You could look at this circuit as voltage divider where

$$V_{th} = V_{oc} = V_{ab} = \frac{240}{(j60 + 36 - j48)} * 36 = \frac{8640}{(36 + j12)}$$

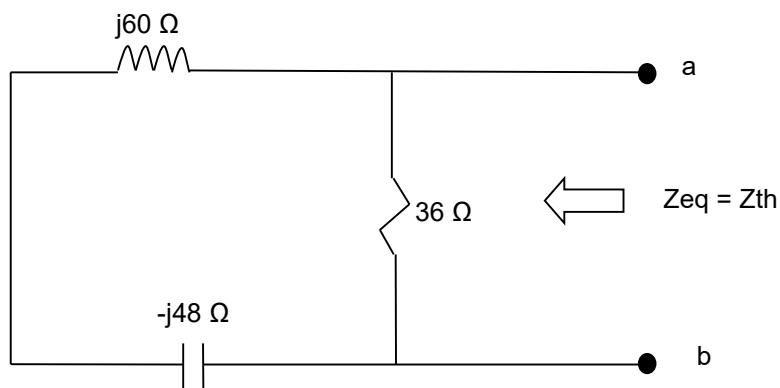
If we rewrite the bottom as a angular form phase then we can divide amplitudes and subtract phases

$$36 + j12 = \sqrt{36^2 + 12^2} \left| \tan^{-1}\left(\frac{12}{36}\right) = 37.95 \right| 18.43^\circ$$

$$V_{th} = \frac{(8640 \angle 0^\circ)}{(37.95 \angle 18.43^\circ)} = \frac{(8640/37.95) \angle (0-18.43)^\circ}{}$$

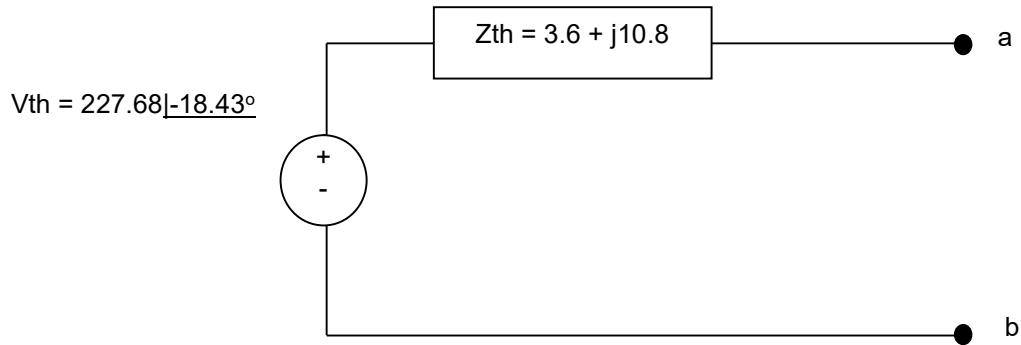
$$V_{th} = 227.668 \angle -18.43^\circ$$

Step 2) Find the  $R_{th} = R_{eq}$  when all independent sources are deactivated

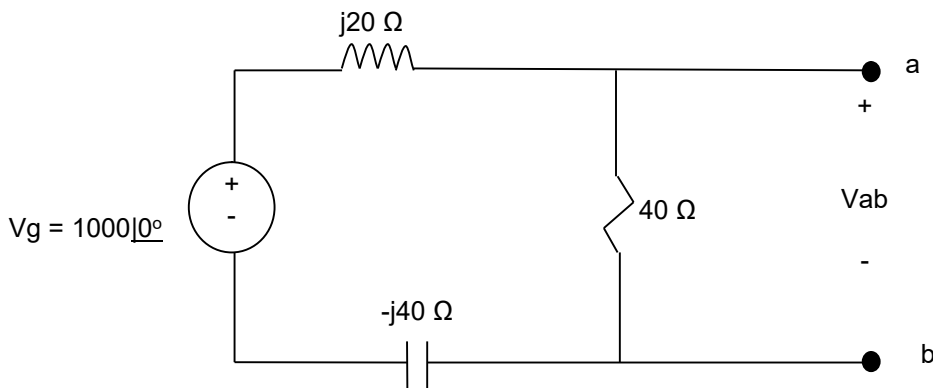


$$Z_{th} = 1 / (1 / (j60 - j48) + 1 / 36) = 3.6 + j10.8$$

Step 3) Draw the Thevenin Equivalent Circuit



11U. Find the Thevenin equivalent circuit with respect to the terminals a, b for the circuit shown in the following figure.

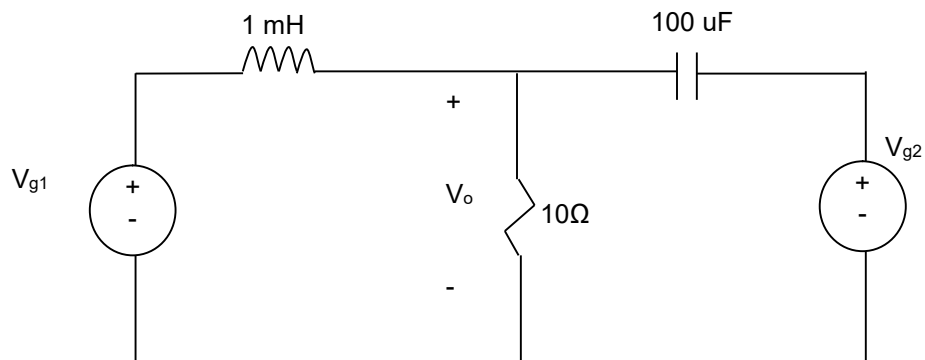


**Solution:**

12S. Use the mesh-current method to find the steady-state expression for the voltage  $v_o(t)$  for the circuit shown below when:

$$v_{g1} = 20 \cos(2000t - 36.87^\circ) \text{ V}$$

$$v_{g2} = 50 \sin(2000t - 16.26^\circ) \text{ V}$$

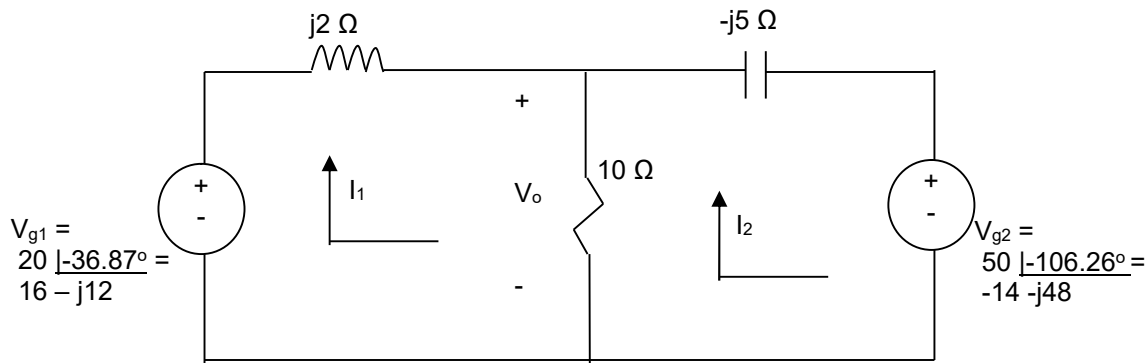


**Solution:**

Step 1) Convert  $v_{g2}$  to Cosine standard form by subtraction  $90^\circ$  from the phase.:  
 $v_{g2} = 50 \sin(2000t - 16.26^\circ) = 50 \cos(2000t - 106.26^\circ)$

Step 2) Convert the circuit to Frequency domain using phasor transformation

Note ( $R \rightarrow R$ ,  $L \rightarrow j\omega L$ ,  $C \rightarrow 1/j\omega C = -j/\omega C$  where  $\omega = 2000$ )



Step 3) Write the KVL equations for the two meshes:

$$\text{Mesh 1} \rightarrow -(16 - j12) + j2I_1 + 10(I_1 - I_2) = 0$$

$$\text{Mesh 2} \rightarrow (-14 - j48) - j5I_2 + 10(I_2 - I_1) = 0$$

From Mesh 1 equation find  $I_2$  in term of  $I_1$ , plug the expression in Mesh 2 equation for  $I_2$

$$I_1 = -6 + j10$$

$$I_2 = -9.6 + j10$$

Step 4) Find the  $V_o$

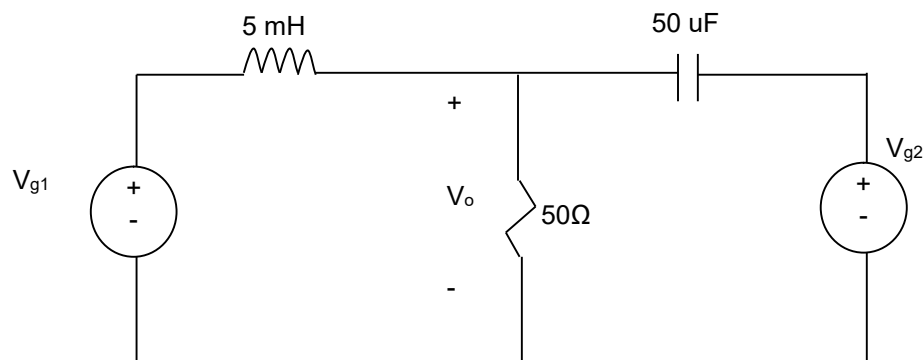
$$V_o = (I_1 - I_2) * 10 = (3.6 * 10) = 36 \angle 0^\circ$$

$$v(t) = 36 \cos(2000t)$$

12U. Use the mesh-current method to find the steady-state expression for the voltage  $v_o(t)$  for the circuit shown below when:

$$v_{g1} = 10 \cos(1000t - 30^\circ) \text{ V}$$

$$v_{g2} = 20 \sin(1000t - 45^\circ) \text{ V}$$



**Solution:**

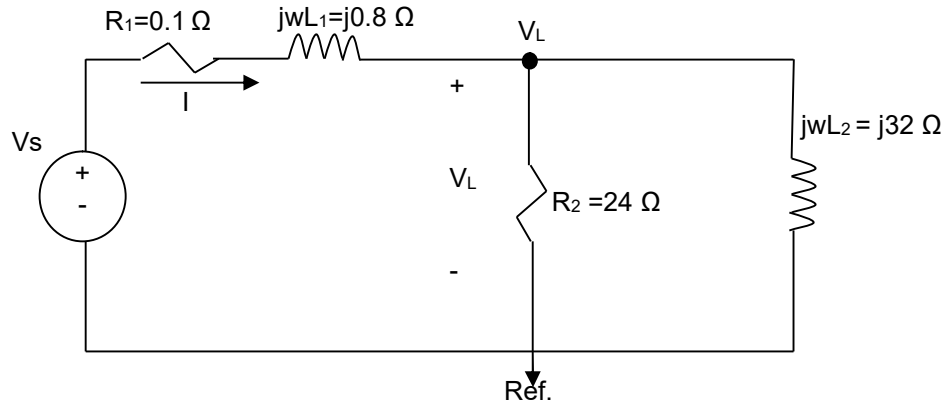
13S. The parameters in the circuit shown below are  $R_1 = 0.1 \Omega$ ,  $\omega L_1 = 0.8 \Omega$ ,  $R_2 = 24 \Omega$ ,  $\omega L_2 = 32 \Omega$  and  $V_L = 240 + j0$ .

a) Calculate the phasor voltage  $V_s$ .

b) Connect a capacitor in parallel with the inductor  $L_2$ , hold  $V_L$  constant, and adjust the capacitor until the magnitude of  $I$  is a minimum. What is the capacitive reactance? What is the Value of  $V_s$ ?

c) Find the value of the capacitive reactance that keeps the magnitude of  $I$  as small as possible while

$$|V_s| = |V_L| = 240 \text{ V.}$$



**Solution:**

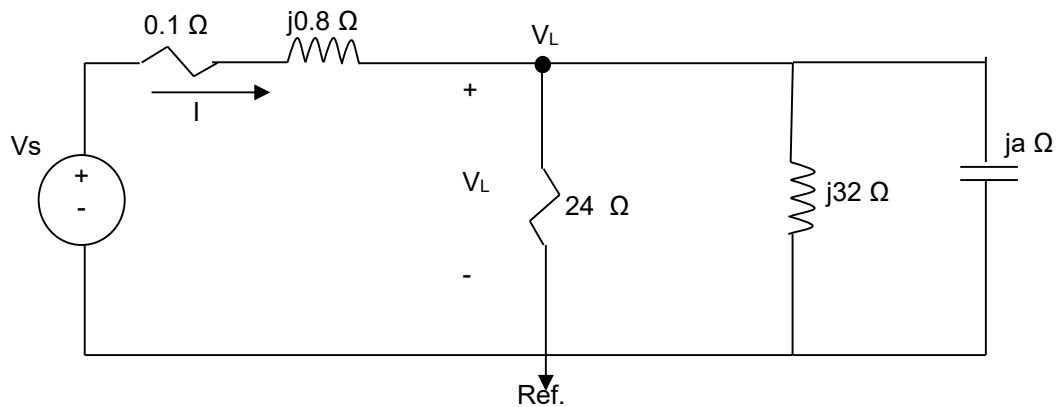
a)  $V_s = ?$

Write KCL at node  $V_L \rightarrow (V_L - V_s)/(R_1 + j\omega L_1) + V_L/R_2 + V_L/j\omega L_2 = 0$

$$(240 - V_s)/(0.1 + j0.8) + 240/24 + 240/j32 = 0$$

$$V_s = (0.1 + j0.8)(240/(0.1 + j0.8) + 10 - j7.5) = 247.35 \angle 1.85^\circ$$

b) Find capacitance reactance =  $a = -1/\omega c$  when  $V_L = 240 \text{ V}$  and magnitude of  $I$  is minimum.



$$I = 240/24 + 240/j32 + 240/ja = 10 - j(7.5 + 240/a)$$

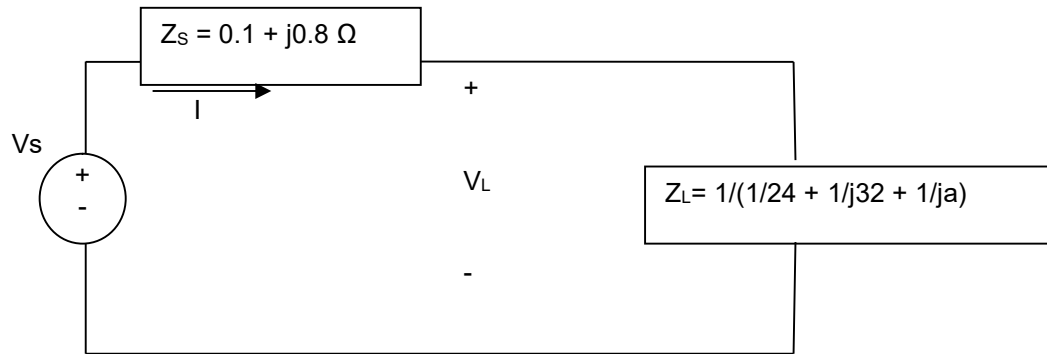
$$\text{magnitude of } I = |I| = \sqrt{10^2 + (7.5 + 240/a)^2}$$

$$\text{Min of } |I| \text{ only when Cap. Cancels out the Inductor so } \rightarrow (7.5 + 240/a)^2 = 0 \rightarrow a = -32$$

Which means  $I = 10 \text{ A}$

$$V_s = (0.1 + j0.8) \cdot 10 + 240 = 241 + j8 = 241.13 \angle 1.9^\circ$$

c) Find capacitance reactance =  $a = -1/\omega c$  when  $|V_L| = |V_s|$  and magnitude of  $I$  is minimum



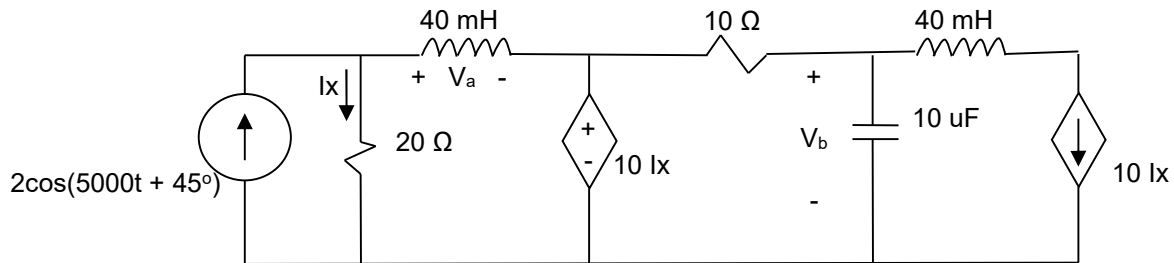
$$|V_L| = |V_s| \left( \frac{Z_L}{Z_L + Z_s} \right)$$

to Meet the condition  $\rightarrow (Z_L / (Z_L + Z_s)) = 1 \rightarrow$  since  $Z_s$  is not 0 then  $Z_L$  must be infinite

$$\text{or the denominator } |Z_L| \text{ of is } 0 \rightarrow |j4 + 3 - j96a| = \sqrt{3^2 + (4 - 96a)^2} = 0 \rightarrow$$

$$(4 - 96a)^2 = -9 \text{ Not Possible}$$

13U. Given the following Circuit:



Find the relationship between  $V_a(t)$  and  $V_b(t)$  when the circuit is in steady state.

**Solution:**