Digital Logic Design - Chapter 1

1S. Convert the following binary numbers to equivalent hexadecimal numbers.

a) $(10001010)_2$
b) $(11110000)_2$
c) $(10000001)_2$
d) $(101001111)_2$

Solution:

			8	4	2	1	8	4	2	1	Hex #
a.	Groups of 4		1	0	0	0	1	0	1	0	
	Conversion			8				A			8A
			8	4	2	1	8	4	2	1	
b.	Groups of 4		1	1	1	1	0	0	0	0	
	Conversion			F				0			F0
			8	4	2	1	8	4	2	1	
c.	Groups of 4		1	0	0	0	0	0	0	1	
	Conversion			8				1			81
			8	4	2	1	8	4	2	1	
d.	Groups of 4	1	0	1	0	0	1	1	1	1	
	Conversion	1		4		.		F			14F

1U. Convert the following binary numbers to equivalent hexadecimal numbers.

a) $(11011010)_2$
b) $(11110011)_2$
c) $(10110101)_2$
d) $(100101111)_2$

Solution:

2S. Convert the following binary numbers to equivalent octal numbers.

a) $(010100)_2$
b) $(111101100)_2$
c) $(100110011)_2$
d) $(011110101111)_2$
e) $(010001110000011)_2$

Solution:

	Octal #											
a.		24	2			4						
			4	2	1	4	2	1				
	Groups of 3		0	1	0	1	0	0				
b.		754	7			5			4			
			4	2	1	4	2	1	4	2	1	
	Groups of 3		1	1	1	1	0	1	1	0	0	

c. 463

	4			6			3		
	4	2	1	4	2	1	4	2	1
Groups of 3	1	0	0	1	1	0	0	1	1

d. 3657

	3			6			5			7		
	4	2	1	4	2	1	4	2	1	4	2	1
Groups of 3	0	1	1	1	1	0	1	0	1	1	1	1

e. 21603

	2			1			6			0			3		
	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
Groups of 3	0	1	0	0	0	1	1	1	0	0	0	0	0	1	1

2U. Convert the following binary numbers to equivalent octal numbers.

a) $(01010101)_2$
b) $(1001011010)_2$
c) $(1000011)_2$
d) $(011111100011)_2$
e) $(010101011100111)_2$

Solution:

3S. Convert the following octal and hex numbers to binary numbers.

a) $(3451)_8$
b) $(65473)_8$
c) $(563451)_8$
d) $(7657.1100)_8$
e) $(BDE)_{16}$
f) $(13F5)_{16}$
g) $(563.4512)_{16}$
h) $(1.1)_{16}$

Solution:

a. $(3451)_8$

3			4			5			1		
4	2	1	4	2	1	4	2	1	4	2	1
0	1	1	1	0	0	1	0	1	0	0	1

b. $(65473)_8$

6			5			4			7			3		
4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
1	1	0	1	0	1	1	0	0	1	1	1	0	1	1

c. $(563451)_8$

5			6			3			4			5			1		
4	2	1	4	2	1	4	2	1	4	2	1	4	2	1	4	2	1
1	0	1	1	1	0	0	1	1	1	0	0	1	0	1	0	0	1

d. $(7657.1100)_8$ Convert by groups of 3: Whole portion of the number :

7			6			5			7		
4	2	1	4	2	1	4	2	1	4	2	1
1	1	1	1	1	0	1	0	1	1	1	1

The fractional portion:

1 1

4	2	1	4	2	1
0	0	1	0	0	1

The entire number is: 111110101111.001001000000

e. $(BDE)_{16}$

B = 11				D = 13				E = 14			
8	4	2	1	8	4	2	1	8	4	2	1
1	0	1	1	1	1	0	1	1	1	1	0

f. $(13F5)_{16}$

1				3				F = 15				5			
8	4	2	1	8	4	2	1	8	4	2	1	8	4	2	1
0	0	0	1	0	0	1	1	1	1	1	1	0	1	0	1

g. $(563.4512)_{16}$ Convert the whole number portion by groups of 4:

$(563)_{16}$

5				6				3			
8	4	2	1	8	4	2	1	8	4	2	1
0	1	0	1	0	1	1	0	0	0	1	1

Convert the fractional portion by groups of 4:

$(.4512)_{16}$

4				5				1				2			
8	4	2	1	8	4	2	1	8	4	2	1	8	4	2	1
0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	0

The entire number is: 010101100011.0100010100010010

h. $(1.1)_{16}$ Convert the number by inspection: $(1.1)_{16} = (1.0001)_{2}$

3U. Convert the following octal and hex numbers to binary numbers.

a) $(2711)_8$
b) $(2CAD)_{16}$
c) $(552612)_8$
d) $(BA2BD)_{16}$

Solution:

4S. Convert the following numbers to their decimal equivalents.

a) $(27431)_8$
b) $(476620)_8$
c) $(1234.567)_8$
d) $(11011110)_8$
e) $(FFFCC)_{16}$
f) $(123430)_{16}$
g) $(E2B4.5E7)_{16}$
h) $(11011110)_{16}$

Solution:

a. $(27431)_8$ $= (o_4\, o_3\, o_2\, o_1\, o_0)_8$
$= (d_4 \times 8^4 + d_3 \times 8^3 + d_2 \times 8^2 + d_1 \times 8^1 + d_0 \times 8^0)_{10}$
$= (2 \times 4096 + 7 \times 512 + 4 \times 64 + 3 \times 8 + 1 \times 1)_{10}$
$= (8192 + 3584 + 256 + 24 + 1)_{10}$
$= (12057)_{10}$

b. $(476620)_8$ $= (o_5\, o_4\, o_3\, o_2\, o_1\, o_0)_8$
$= (d_5 \times 8^5 + d_4 \times 8^4 + d_3 \times 8^3 + d_2 \times 8^2 + d_1 \times 8^1 + d_0 \times 8^0)_{10}$
$= (4 \times 32{,}768 + 7 \times 4096 + 6 \times 512 + 6 \times 64 + 2 \times 8 + 0 \times 1)_{10}$
$= (131{,}072 + 28{,}672 + 3072 + 384 + 16 + 0)_{10}$
$= (163{,}216)_{10}$

c. $(1234.567)_8$ $= (o_4\, o_3\, o_2\, o_1\, o_0\, .\, o_{-1}\, o_{-2}\, o_{-3})_8$
$= (d_3 \times 8^3 + d_2 \times 8^2 + d_1 \times 8^1 + d_0 \times 8^0 + d_{-1} \times 8^{-1} + d_{-2} \times 8^{-2} + d_{-3} \times 8^{-3})_{10}$
$= (1 \times 512 + 2 \times 64 + 3 \times 8 + 4 \times 1 + 5 \times 0.125 + 6 \times .016 + 7 \times .002)_{10}$
$= (512 + 128 + 24 + 4 + 0.625 + .094 + .014)_{10}$
$= (668.732)_{10}$

d. $(11011110)_8$ $= (o_7\, o_6\, o_5\, o_4\, o_3\, o_2\, o_1\, o_0)_8$
$= (d_7 \times 8^7 + d_6 \times 8^6 + d_5 \times 8^5 + d_4 \times 8^4 + d_3 \times 8^3 + d_2 \times 8^2 + d_1 \times 8^1 + d_0 \times 8^0)_{10}$
$= (1 \times 2{,}097{,}152 + 1 \times 262{,}144 + 0 \times 32{,}768 + 1 \times 4096 +$
$1 \times 512 + 1 \times 64 + 1 \times 8 + 0 \times 1)_{10}$
$= (2{,}363{,}976)_{10}$

e. $(FFFCC)_{16}$ $= (h_4\, h_3\, h_2\, h_1\, h_0)_{16}$
$= (h_4 \times 16^4 + h_3 \times 16^3 + h_2 \times 16^2 + h_1 \times 16^1 + h_0 \times 16^0)_{10}$
$= (15 \times 65{,}536 + 15 \times 4096 + 15 \times 256 + 12 \times 16 + 12 \times 1)_{10}$
$= (983{,}040 + 61{,}440 + 3840 + 192 + 12)_{10}$
$= (1{,}048{,}524)_{10}$

f. $(123430)_{16}$ $= (h_4\, h_3\, h_2\, h_1\, h_0)_{16}$
$= (h_5 \times 16^5 + h_4 \times 16^4 + h_3 \times 16^3 + h_2 \times 16^2 + h_1 \times 16^1 + h_0 \times 16^0)_{10}$
$= (1 \times 1{,}048{,}576 + 2 \times 65{,}536 + 3 \times 4096 + 4 \times 256 + 3 \times 16 + 0 \times 1)_{10}$
$= (1{,}048{,}576 + 131{,}072 + 12{,}288 + 1024 + 48 + 0)_{10}$
$= (8192 + 3584 + 256 + 24 + 1)_{10}$
$= (1{,}193{,}008)_{10}$

g. $(E2B4.5E7)_{16}$ $= (h_4\, h_3\, h_2\, h_1\, h_{0.}\, h_{-1}\, h_{-2}\, h_{-3})_{16}$
$= (h_4 \times 16^4 + h_3 \times 16^3 + h_2 \times 16^2 + h_1 \times 16^1 + h_0 \times 16^0 + h_{-1} \times 16^{-1} + h_{-2} \times 16^{-2} + h_{-3} \times 16^{-3})_{10}$
$= (14 \times 4096 + 2 \times 256 + 11 \times 16 + 4 \times 1 + 5 \times .063 + 14 \times .004 + 7 \times 2.44\text{X } 10\text{^-}4)_{10}$
$= (57344 + 512 + 176 + 4 + .313 + .055 + .002)_{10}$
$= (58036.369)_{10}$

h. $(11011110)_{16}$ $= (h_7\, h_6\, h_5\, h_4\, h_3\, h_2\, h_1\, h_0)_{16}$
$= (d_7 \times 16^7 + d_6 \times 16^6 + d_5 \times 16^5 + d_4 \times 16^4 + d_3 \times 16+^3 + d_2 \times 16^2 + d_1 \times 16^1 + d_0 \times 16^0)_{10}$
$= (1 \times 268435456 + 1 \times 16777216 + 0 \times 16^5 + 1 \times 16^4 + 1 \times 16^3 +$
$1 \times 16^2 + d_1 \times 8^1 + d_0 \times 8^0)_{10}$
$= (285{,}282{,}576)_{10}$

4U. Convert the following numbers to their decimal equivalents.

a) $(27431)_8$
b) $(476620)_{16}$
c) $(11011110)_2$

Solution:

5S. Obtain the decimal values for the following binary numbers expressed in a 2's complement representation (in each case the most significant bit is the sign bit).

a. $(0110110)_{RC}$
b. $(01011010111)_{RC}$
c. $(101100101)_{RC}$

Solution:

Notes:

- Sign bit: 1 for negative and 0 for positive
- Magnitude : bits 0 to (n-1) where n is word size
- Only when sign bit is 1, 2's complemented must be performed to find the positive magnitude

a. $(0110110)_{RC}$ → $+(0110110)_2 = +54$
b. $(01011010111)_{RC}$ → $+(01011010111)_2 = +727$
c. $(101100101)_{RC}$ → $-(010011011)_2 = -155$

5U. Obtain the decimal values for the following binary numbers expressed in a 2's complement representation (in each case the most significant bit is the sign bit).

a. $(1111110)_{RC}$
b. $(01111011111)_{RC}$
c. $(1000001011010)_{RC}$

Solution:

6S. Carry out the following arithmetic addition operations in both decimal and RC representation. Use a word size of 8 bits for the numbers expressed in RC representation.

a. 6 +(-3)
b. 95 + 27
c. 101 + (-46)
d. 39 + (-17)

Solution:

a. 6 +(-3)

6 – 3 = 3

6

SB	64	32	16	8	4	2	1
0	0	0	0	0	1	1	0

3

	SB	64	32	16	8	4	2	1
	0	0	0	0	0	0	1	1
2's comp -3	1	1	1	1	1	1	0	1

carry	1	1	1	1	1	1	0	0	
6		0	0	0	0	0	1	1	0
-3		1	1	1	1	1	1	0	1
3		0	0	0	0	0	0	1	1

b. 95 + 27 = 122

95			95					
-64	128	64	32	16	8	4	2	1
31	0	1	0	1	1	1	1	1
-16								
15								
-8								
7								
-4								
3								
-2								
1								
-1								

27			27					
-16	128	64	32	16	8	4	2	1
11	0	0	0	1	1	0	1	1
-8								
3								
-2								
1								
-1								

carry		0	0	1	1	1	1	1	
	95	0	1	0	1	1	1	1	1
	27	0	0	0	1	1	0	1	1
	122	0	1	1	1	1	0	1	0

c. 101 + (-46)

101			101					
-64	SB	64	32	16	8	4	2	1
37	0	1	1	0	0	1	0	1
-32								
5								
-4								
1								
-1								

46			46					
-32	SB	64	32	16	8	4	2	1
14	0	0	1	0	1	1	1	0
-8								
6								
-4								
2								
-2								

46			46					
	SB	64	32	16	8	4	2	1

	46	0	0	1	0	1	1	1	0	
2's comp		1	1	0	1	0	0	1	0	
carry		1	1	0	0	0	0	0	0	
	101		0	1	1	0	0	1	0	1
	-46		1	1	0	1	0	0	1	0
	55		0	0	1	1	0	1	1	1

d. 39 + (-17)

39			39					
-32	SB	64	32	16	8	4	2	1
7	0	0	1	0	0	1	1	1
-4								
3								
-2								
1								
-1								

17			17					
-16	SB	64	32	16	8	4	2	1
1	0	0	0	1	0	0	0	1
-1								

	17			17						
		SB	64	32	16	8	4	2	1	
	17	0	0	0	1	0	0	0	1	
2's comp		1	1	1	0	1	1	1	1	
carry		1	1	1	0	1	1	1	1	
	39		0	0	1	0	0	1	1	1
	-17		1	1	1	0	1	1	1	1
	22		0	0	0	1	0	1	1	0

6U. Carry out the following arithmetic addition operations in both decimal and RC representation. Use a word size of 8 bits for the numbers expressed in RC representation.

a. 123 +(-125)
b. (-105) + 127
c. (-101) + 98
d. 119 + (-17)

Solution:

7S. Determine the minimum number of bits that are required to represent all the characters on a keyboard that has the following number of keys.

a. 9
b. 16
c. 22
d. 36
e. 104

Solution:

a. 9

Number of bits = ln(number of keys) / ln(2)
= ln(9) / ln(2)
= 2.197 / .693
= 3.17
= 4 bits

b. 16

Number of bits = ln(number of keys) / ln(2)
= ln(16) / ln(2)
= 2.773/.693
= 4 bits

c. 22

Number of bits = ln(number of keys) / ln(2)
= ln(22) / ln(2)
= 3.091/.693
= 4.459
= 5 bits

d. 36

Number of bits = ln(number of keys) / ln(2)
= ln(36)/ln(2)
= 3.584/.693
= 5.170
= 6 bits

e. 104

Number of bits = ln(number of keys) / ln(2)
= ln(104)/ln(2)
= 4.644/.693
= 6.700
= 7 bits

7U. Determine the minimum number of bits required to uniquely represent each art work in a gallery with as many as:

a) 2520 art pieces
b) 4098 art pieces
c) 23109 art pieces

Solution:

8S. Look up and record the ASCII character for the following binary bit patterns.

a. 1111001
b. 1010110
c. 0000011
d. 1011001
e. 1100110

Solution:

a. 1111001 ASCII character: y
b. 1010110 ASCII character: V
c. 0000011 ASCII character: ETX
d. 1011001 ASCII character: Y

e. 1100110 ASCII character: f

8U. Write down the ASCII codes (in Dec.) for the characters in the string excluding the quotation marks for:

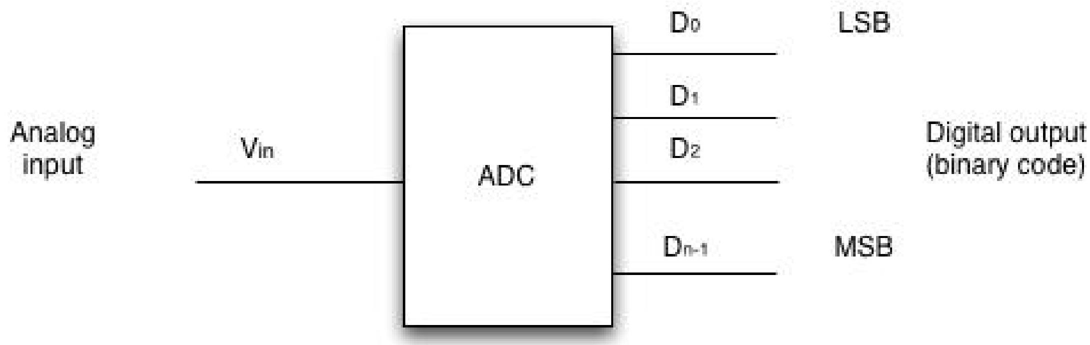
a. "Hello, Hope you are doing well!"
b. "This is a wonderful course!"

Solution:

9S. A block diagram for an analog-to-digital converter or ADC is shown below. This device converts an analog signal (a voltage) supplied to its input into a binary code at its output. The resolution of each bit, also called the resolution of the LSB (the voltage value), the full-scale range of the input voltage V_{in}, and the number of output bits *n* of the ADC are related as follows:

$$\text{Resolution of LSB} = V_{in} / (2^n-1)$$

Given the following requirements, find the minimum number of output bits for each ADC.



a. Resolution of LSB <= 0.25 volts and a full-scale voltage range of 5 volts
b. Resolution of LSB <= 70 millivolts and a full-scale voltage range of 10 volts
c. Resolution of LSB <= 8 millivolts and a full-scale voltage range of 12 volts
d. Resolution of LSB <= 3/4 millivolts and a full-scale voltage range of 12 volts

Solution:

a. Resolution of LSB <= 0.25 volts and a full-scale voltage range of 5 volts

$2^n >= V_{FSR}$ / Resolution of LSB + 1
>= 5 / .25 + 1
32 >= 21
n = 5

b. Resolution of LSB <= 70 millivolts and a full-scale voltage range of 10 volts

$2^n >= V_{FSR}$ / Resolution of LSB +1
>= 10 / .07 + 1
256 >= 143.9
n = 8

c. Resolution of LSB <= 8 millivolts and a full-scale voltage range of 12 volts

$2^n >= V_{FSR}$ / Resolution of LSB + 1
>= 12 / .008 + 1
2048 >= 1501
n = 11

d. Resolution of LSB <= 3/4 millivolts and a full-scale voltage range of 12 volts

$$2^n \geq V_{FSR} / \text{Resolution of LSB} + 1$$
$$\geq 12 / .00075 + 1$$
$$16384 \geq 16001$$
$$n = 14$$

9U. We have been asked to determine the number of output bits required on an analog-to-digital converter for each of the following cases:

a. Resolution of LSB <= 4 millivolts and a full-scale voltage range of 24 volts
b. Full-scale voltage range of 25 v with resolution of 1 mv.
c. Range from low voltage of -20 v to high of 30 v and a unique output for every 15 mv.

Solution: