

Fundamentals of Electrical Circuits - Chapter 10

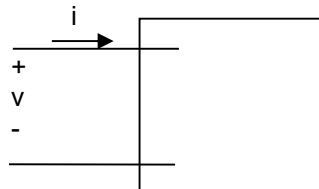
1S. The following sets of values for v and i pertain to the circuit seen in following figure. For each set of values calculate P & Q and state whether the circuit inside the box is absorbing or delivering (1) average power and (2) magnetizing vars.

a) $v = 100 \cos(\omega t + 50^\circ)$ V, $i = 10 \cos(\omega t + 15^\circ)$ A.

b) $v = 40 \cos(\omega t - 15^\circ)$ V, $i = 20 \cos(\omega t + 60^\circ)$ A.

c) $v = 400 \cos(\omega t + 30^\circ)$ V, $i = 10 \sin(\omega t + 240^\circ)$ A.

d) $v = 200 \sin(\omega t + 250^\circ)$ V, $i = 5 \cos(\omega t + 40^\circ)$ A.



Solution:

We have

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \text{ If Positive is absorbing; if negative is generating (Passive Convention)}$$

$$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$$

a)
$$P = \frac{1}{2} (100)(10) \cos(50 - 15) = 409.58 \text{ W (abs)}$$

$$Q = \frac{1}{2} (100)(10) \sin(50 - 15) = 286.79 \text{ VAR (abs)}$$

b)
$$P = \frac{1}{2} (40)(20) \cos(-15 - 60) = 103.52 \text{ W (abs)}$$

$$Q = \frac{1}{2} (40)(20) \sin(-15 - 60) = -386.37 \text{ VAR (gen)}$$

c) $v = 400 \cos(\omega t + 30^\circ)$ V

$$i = 10 \sin(\omega t + 240^\circ) \text{ A} \rightarrow I = 10 \cos(\omega t + 240 - 90) = 10 \cos(\omega t + 150)$$

$$P = \frac{1}{2} (400)(10) \cos(30 - 150) = -1,000 \text{ W (gen)}$$

$$Q = \frac{1}{2} (400)(10) \sin(30 - 150) = -1,732.05 \text{ VAR (gen)}$$

d) $v = 200 \sin(\omega t + 250^\circ) = 200 \cos(\omega t + 250 - 90) = 200 \cos(\omega t + 160)$

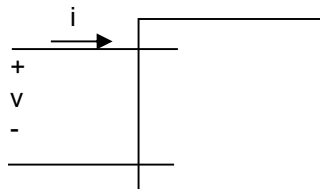
$$i = 5 \cos(\omega t + 40^\circ) \text{ A.}$$

$$P = \frac{1}{2} (200)(5) \cos(160 - 40) = -250 \text{ W (generating)}$$

$$Q = \frac{1}{2} (200)(5) \sin(160 - 40) = +433.01 \text{ VAR (Consuming)}$$

1U. The following sets of values for v and i pertain to the circuit seen in following figure. For each set of values calculate P & Q and state whether the circuit inside the box is absorbing or delivering (1) average power and (2) magnetizing vars.

- $v = 100 \cos(\omega t + 90^\circ)$ V, $i = 20 \sin(\omega t - 30^\circ)$ A.
- $v = 20 \sin(\omega t - 45^\circ)$ V, $i = 60 \cos(\omega t + 30^\circ)$ A.
- $v = 45 \cos(\omega t - 120^\circ)$ V, $i = 15 \sin(\omega t + 240^\circ)$ A.
- $v = 100 \cos(\omega t + 250^\circ)$ V, $i = 5 \cos(\omega t + 40^\circ)$ A.



2S. Given $I_{\text{eff}} = 10.6$ A, $V_{\text{eff}} = 110$ V and the current leads voltage by $\pi/4$, determine the value of real and reactive power.

Solution:

$$\text{Real Power} = I_{\text{eff}} V_{\text{eff}} \cos(\theta_v - \theta_i) = (10.6)(110)\cos(-\pi/4) = -824.5 \text{ W}$$

$$\text{Reactive Power} = I_{\text{eff}} V_{\text{eff}} \sin(\theta_v - \theta_i) = (10.6)(110)\sin(-\pi/4) = -824.5 \text{ VAR}$$

2U. Given $I_{\text{eff}} = 5.8$ A, $V_{\text{eff}} = 240$ V and the voltage leads current by $\pi/6$, determine the value of real and reactive power.

Solution:

3S. You are drying your hair, sitting under a sunlamp and watching a basketball game on television. At the same time your housemate is vacuuming the rug in your air-conditioned house. All these appliances are supplied from a 120 V branch circuit protected by a 15 A circuit breaker with the following power requirements:

Product	Average Power usage (Watts)
Hair dryer	600
Television (tub-type)	240
Vacuum Cleaner	630
Electric Car	279
Air condition (room)	860

Use the above information to answer the following questions:

- Will the breaker interrupt the game?
- Will you be able to watch television if you turn off the sunlamp and turn off the vacuum cleaner?

Solution:

a) Total Power being used = $P\{\text{hair dryer}\} + P\{\text{sunlamp}\} + P\{\text{Television}\} + P\{\text{Vacuum}\} + P\{\text{Air Condition}\}$
 $= 600 + 240 + 630 + 279 + 860 = 2609 \text{ W}$

$$I_{\text{eff}} = \text{Total Power} / V_{\text{eff}} = 2609 / 120 = 21.74 \text{ A}$$

Since I_{eff} needed is more than Circuit Breaker (15 A) would allow, the game will be interrupted.

b) Total Power being used = $P\{\text{hair dryer}\} + P\{\text{Television}\} + P\{\text{Air Condition}\}$
 $= 600 + 240 + 860 = 1700 \text{ W}$

$$I_{\text{eff}} = \text{Total Power} / V_{\text{eff}} = 1700 / 120 = 14.2 \text{ A}$$

Since I_{eff} needed is less than Circuit Breaker (15 A) would allow, the game will not be interrupted.

3U. You are working on your computer while the television is on, and heating your dinner in the microwave. At the same time your housemate is vacuuming the rug in your air-conditioned house. All these appliances are supplied from a 120 V branch circuit protected by a 20 A circuit breaker with the following power requirements:

Product	Average Power usage (Watts)
Microwave	2000
Television	120
Vacuum Cleaner	700
Light	150
Air condition	1500
Computer System	160

Use the above information to answer the following questions:

- Will you exceed the circuit breaker's capacity and loose all your unsaved work on the computer?
- Will you be able to continue working on computer if you turn off the microwave and vacuum cleaner?

Solution:

4S. A personal Computer with a monitor and keyboard requires 40 W at 115 V (rms).

- Calculate the rms value of the current carried by its power cord.
- A laser printer for the personal computer in (a) is rated at 90 W at 115 V(rms). If this printer is plugged into the same wall outlet as the computer what is the rms value of the current drawn from the outlet.

Solution:

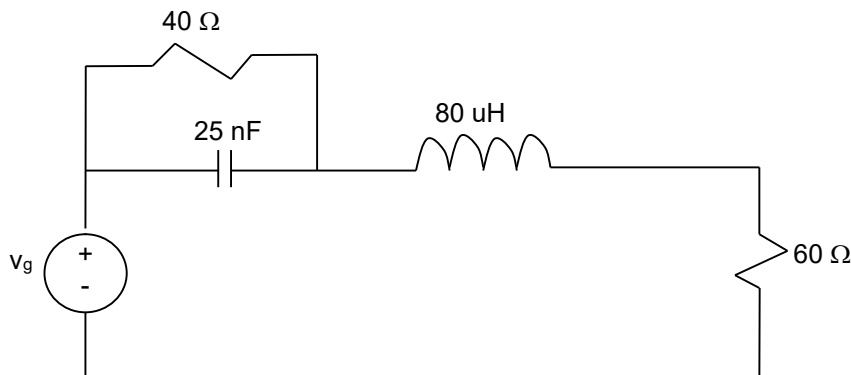
- $I_{\text{eff PC}} = P / V_{\text{eff}} = 40 / 115 = .35 \text{ A}$
- $I_{\text{eff printer}} = P / V_{\text{eff}} = 90 / 115 = .78 \text{ A}$
 $I_{\text{eff outlet}} = I_{\text{eff PC}} + I_{\text{eff printer}} = .35 + .78 = 1.13 \text{ A}$

4U. A Smart phone charger uses maximum of 100 mW at 115 V (rms).

- Calculate the rms value of the current carried by its power cord.
- A bluetooth-enabled laser printer is connected to the same outlet while the smart phone is being charged. Printer is rated at 90 W at 115 V(rms). What is the rms value of the current drawn from the outlet.

Solution:

5S. The following Circuit is powered by $v_g = 40 \cos 10^6 t \text{ V}$.



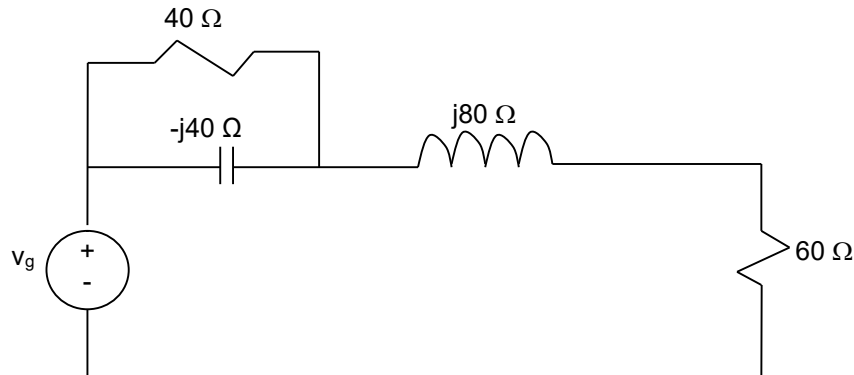
- Find the average power, the reactive power and the apparent power supplied by the voltage source in the following circuit (Steady State).
- Check your answer in (a) by showing magnitudes of P_{dev} and ΣP_{abs} are equal.

c) Check your answer in (a) by showing magnitudes of Q_{dev} and $\Sigma Q_{abs.}$ are equal.

Solution:

a)

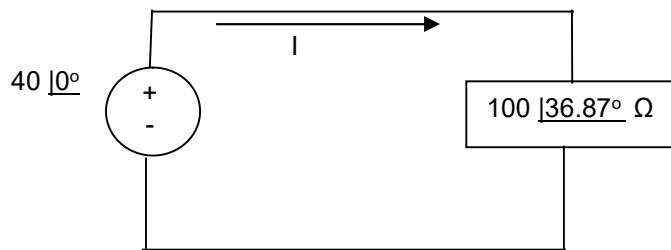
Step 1) Redraw the circuit in Frequency domain ($R \rightarrow R$, $C \rightarrow -j/wC$, $L \rightarrow jwL$ where $w = 10^6$)



Step 2) Find

$$Z_{eq} \text{ seen by the supply} = (40 \parallel -j40) + j80 + 60 = 80 + j60 = \sqrt{80^2 + 60^2} \angle \tan^{-1} 60/80$$

$$Z_{eq} = 100 \angle 36.87^\circ \Omega$$



$$I = \frac{V}{Z} = \frac{40 \angle 0^\circ}{100 \angle 36.87^\circ} = .4 \angle -36.87^\circ \text{ A}$$

$$P = -\frac{1}{2}(40)(.4) \cos(0 - (-36.87)) = -6.4 \text{ W (del)}$$

$$Q = -\frac{1}{2}(40)(.4) \sin(0 - (-36.87)) = -4.8 \text{ VAR (del)}$$

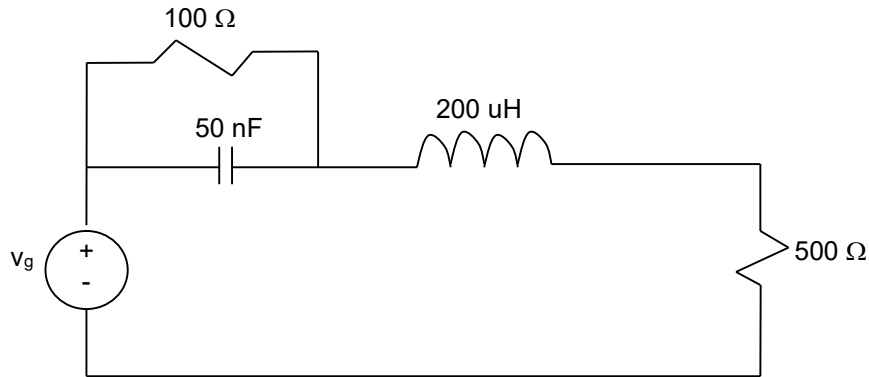
$$\text{Complex Power} = S = P + jQ = -6.4 - j4.8 \text{ VA}$$

$$\text{Apparent Power} = |S| = \sqrt{6.4^2 + 4.8^2} = 8 \text{ VA}$$

b & c) Now the absorbed side:

$$S = |I_{eff}|^2 Z = (1/2) |I_m|^2 Z = (1/2)(.4^2)(80 + j60) = 6.4 + j4.8 = P + jQ \rightarrow P=6.4 \text{ W}; Q=4.8 \text{ VA}$$

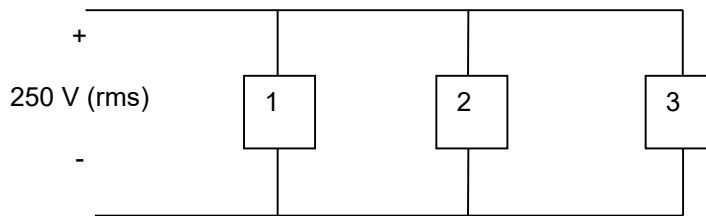
5U. The following Circuit is powered by $v_g = 60 \cos 2000\pi t$ V.



- Find the average power, the reactive power and the apparent power supplied by the voltage source in the following circuit if.
- Check your answer in (a) by showing magnitudes of P_{dev} and $\Sigma P_{abs.}$ are equal.
- Check your answer in (a) by showing magnitudes of Q_{dev} and $\Sigma Q_{abs.}$ are equal.

Solution:

6S. Three loads are connected in parallel across a 250 V (rms) line, as shown in the following figure. Load 1 absorbs 16 kW and 18 kVAR. Load 2 absorbs 10 kVA at 0.6 pf lead. Load 3 absorbs 8 kW at unity power factor.



- Find the impedance that is equivalent to three parallel loads.
- Find the power factor of the equivalent load as seen from the line's input terminals.

Solution:

a) Given:

$$V_{rms} = V_{eff} = 250 \text{ V}$$

$$S_1 = 16 + j18 \text{ kVA}$$

$$|S_2| = 10 \text{ kVA at pf}_2 = \cos(\theta_v - \theta_i) = 0.6 \text{ leading} \rightarrow (\theta_v - \theta_i) = -\cos^{-1}(0.6) = -53^\circ$$

$$P_2 = |S_2| \cos(\theta_v - \theta_i) = 10 * \cos(-53) = 6 \text{ kW}$$

$$Q_2 = |S_2| \sin(\theta_v - \theta_i) = 10 * \sin(-53) = -8 \text{ kVAR}$$

$$S_2 = P + jQ = 6 - j8 \text{ kVA}$$

$$|S_3| = 8 \text{ kW at pf} = 1$$

$$S_3 = 8 \text{ kVA}$$

$$S_{total} = S_1 + S_2 + S_3 = 30 + j10 \text{ kVA}$$

using

$$S = \frac{|V_{eff}|^2}{Z^*} \Rightarrow Z^* = \frac{|V_{eff}|^2}{S}$$

$$Z^* = \frac{250^2}{1000(30+j10)} = \frac{62.5}{\sqrt{30^2 + 10^2} \angle \tan^{-1}(10/30)} = \frac{62.5}{31.62 \angle 18.43^\circ} = 1.98 \angle -18.43^\circ = 1.89 - j.63$$

$$Z = 1.88 + j.63 = 1.98 \angle 18.43^\circ$$

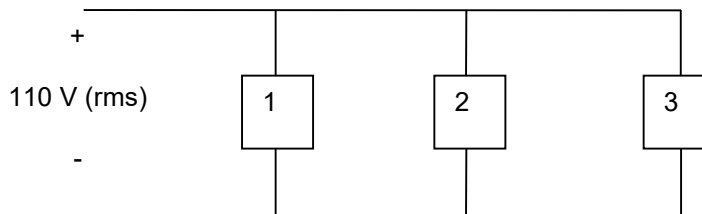
b) $S_{total} = 30 + j10$ kVA

Power factor angle = $(\theta_v - \theta_i) = \tan^{-1}(10/30) = 18.43^\circ$

Power factor = $\cos(\text{power factor angle}) = \cos(18.43^\circ) = .9487$

Lagging since $(\theta_v - \theta_i) > 0$ meaning or Current lags the voltage

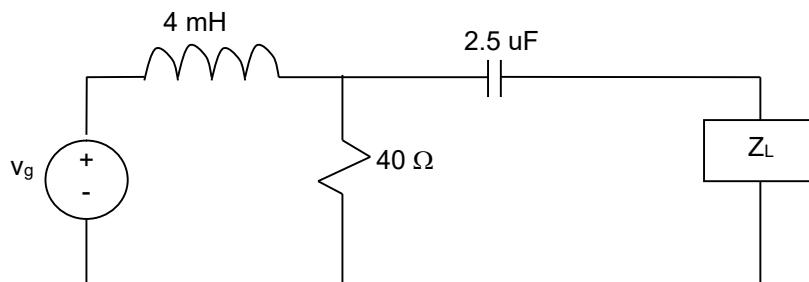
6U. Three loads are connected in parallel across a 110 V (rms) line, as shown in the following figure. Load 1 absorbs 5 kW and 25 kVAR. Load 2 absorbs 20 kVA at 0.5 pf lagging. Load 3 absorbs 15 kW at unity power factor.



- Find the impedance that is equivalent to three parallel loads.
- Find the power factor of the equivalent load as seen from the line's input terminals.

Solution:

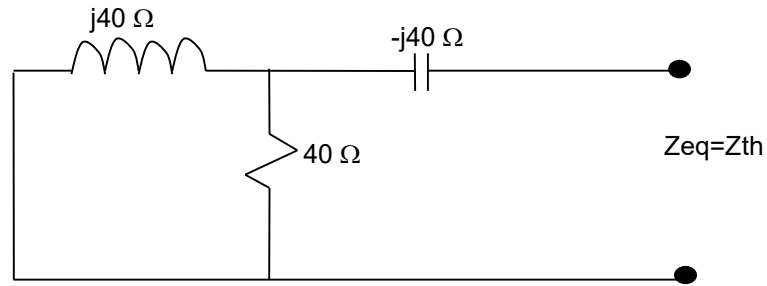
7S. a) Determine the load impedance for the following circuit that will result in maximum average power being transferred to the load if $\omega = 10$ krad/s.



- Determine the maximum average power if $v_g = 120 \cos 10,000t$ V.

Solution:

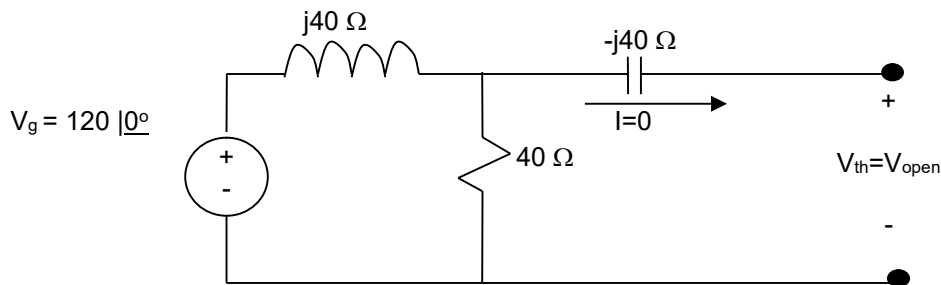
- First need to find the thevenin equivalent with respect to the load (Z_L)
 Step 1) Redraw the circuit in Frequency domain ($R \rightarrow R$, $C \rightarrow -j/\omega C$, $L \rightarrow j\omega L$ where $\omega = 10^4$)
 Step 2) Deactivate the supply ($v=0$ or short) to find the $Z_{eq} = Z_{th}$



$$Z_{th} = (40 \parallel j40) - j40 = 20 - j20 = 28.3 \angle -45^\circ \Omega$$

Maximum power is transferred when $Z_L = Z_{th}^* = 20 + j20 = 28.3 \angle +45^\circ \Omega$

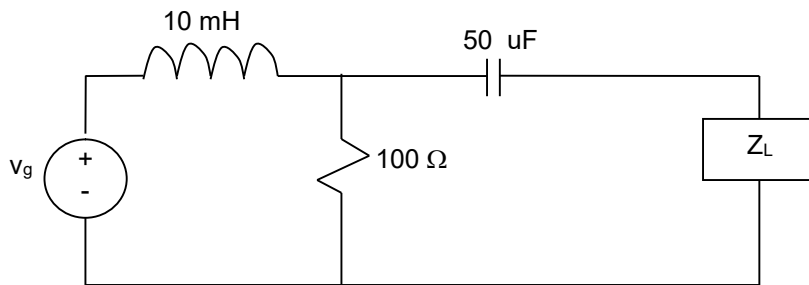
b) Now we have to first find the find $V_{open} = V_{th}$



$$V_{th} = \{120 / (40 + j40)\} 40 = 60 - j60 = 84.8 \angle -45^\circ \text{ V}$$

$$P_{max} = (V_{eff})^2 / (4R_L) = (1/8) (V_m^2) / (R_L) = (1/8)(84.8)^2 / (20) = 44.9 \text{ W}$$

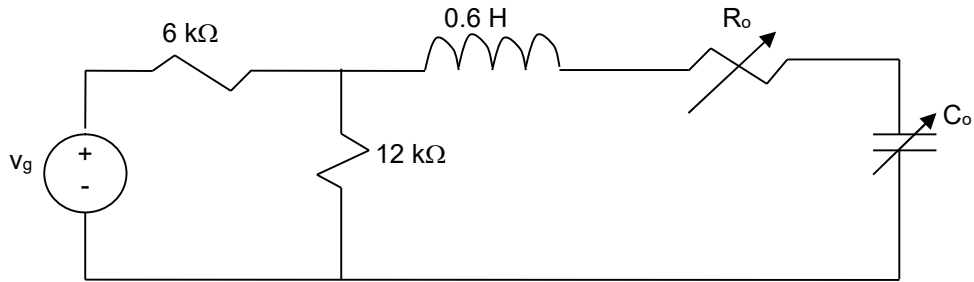
7U. a) Determine the load impedance for the following circuit that will result in maximum average power being transferred to the load if $\omega = 20 \text{ krad/s}$.



b) Determine the maximum average power if $v_g = 50 \cos 20,000t \text{ V}$.

Solution:

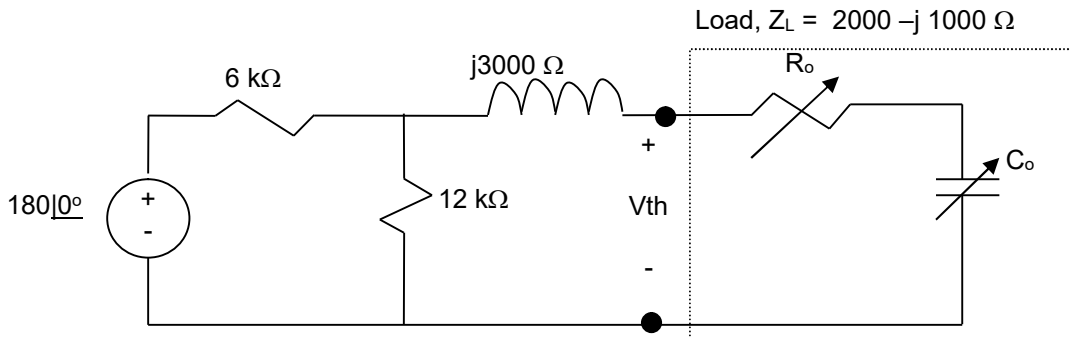
8S. The peak amplitude of the sinusoidal voltage source in the circuit shown in following circuit is 180 V, and its frequency is 5000 rad/sec. The load resistor can be varied from 0 to 4000Ω, and the load capacitor can be varied from 0.1 to 0.5 uF.



- Calculate the average power delivered to the load when $R_o = 2000 \Omega$ and $C_o = 0.2 \mu\text{F}$.
- Determine the settings of R_o and C_o that will result in the most average power being transferred to R_o .
- What is the most average power in (b)? Is it greater than power in (a)?
- If there are no constraints on R_o and C_o , what is the maximum average power that can be delivered to a load?
- What are the values of R_o and C_o for the condition of (d)?
- Is the average power calculated in (d) larger than that calculated in (c)?

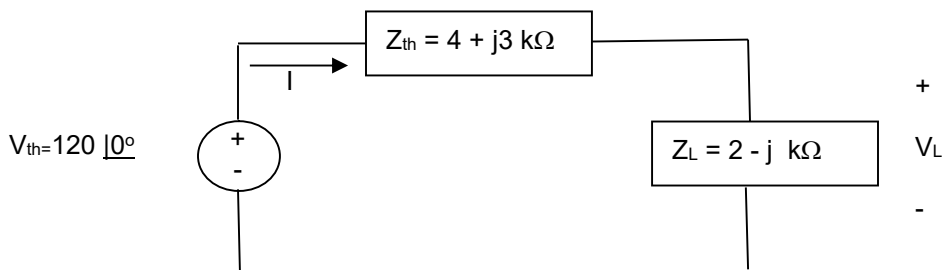
Solution:

- First need to find the thevenin equivalent with respect to the load (Z_L)
 - Redraw the circuit in Frequency domain ($R \rightarrow R, C \rightarrow -j/\omega C, L \rightarrow j\omega L$ where $\omega = 5,000$)
 - Deactivate the supply ($v=0$ or short) to find the $Z_{eq} = Z_{th}$



$$Z_{th} = (6 \parallel 12) + j3 = 4 + j3 \text{ k}\Omega = 5 \angle 36.87^\circ \text{ k}\Omega$$

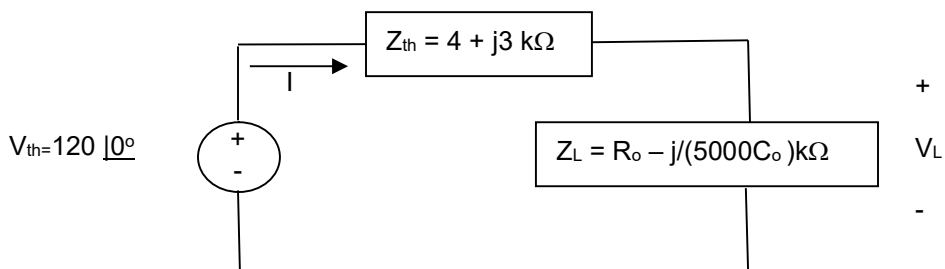
$$V_{th} = \left(\frac{180}{6+12} \right) 12 = 120 \angle 0^\circ \text{ V}$$



$$I = (120) \left(\frac{1}{4 + j3 + 2 - j} \right) = \frac{120}{6 + 2j} = \frac{120}{\sqrt{36+4} \angle \tan^{-1} 2/6} = 18.97 \angle -18.43^\circ \text{ mA}$$

$$P = \text{average power} = |I_{eff}|^2 R_L = (1/2) |I_m|^2 R_L = (1/2)(18.97 \times 10^{-3})^2 (2000) = 360 \text{ mW}$$

b) We can leverage last section since the Thevenin equivalent will be the same only the load is different



The condition for maximum power transfer is $Z_{th} = Z_L^*$ therefore

$$4 + j3 = R_o + j/5000C_o \rightarrow R_o = 4 \text{ k}\Omega$$

&

$$1/(5000)C_o = 3 \text{ k}\Omega \rightarrow C_o = 66.67 \text{ nf (Violates 0.1-0.5 uF restriction)}$$

c) The restrictions that the load resistor can be varied from 0 to 4000Ω, and the load capacitor can be varied from 0.1 to 0.5 uF Leads to setting :

$$Z_L = 4000 -j/(5000)(.1E-6) \rightarrow Z_L = 4 -j2 \text{ k}\Omega$$

$$I = (120) \left(\frac{1}{4 + j3 + 4 - j2} \right) = \frac{120}{8 + 1j} = \frac{120}{\sqrt{64 + 1} \angle \tan^{-1} 1/8} = 14.88 \angle -7.13^\circ \text{ mA}$$

P = average power = $|I_{eff}|^2 R_L = (1/2) |I_m|^2 R_L = (1/2)(14.88 \times 10^{-3})^2 (4000) = 443 \text{ mW}$ which is greater than 360 mW in part (a)

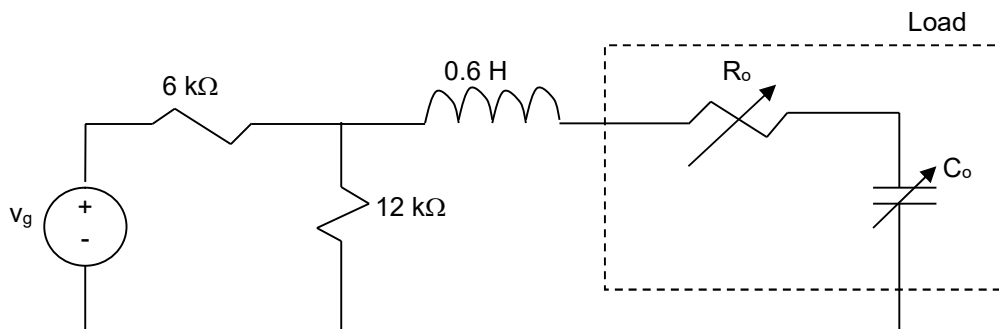
d) When there are no constraint we can use the values from b to find maximum average power transfer

$$P_{max} = (1/8)(V_{m-th}^2/R_L) = (1/8)(120^2)/(1/4000) = 450 \text{ mW}$$

e) Same as derived in part b.

f) Yes. 450 mW > 443 mW as expected.

8U. The peak amplitude of the sinusoidal voltage source in the circuit shown in following circuit is 2 V, and its frequency is $20,000\pi$ rad/sec. The load resistor can be varied from 1 to 40 KΩ, and the load capacitor can be varied from 0.2 to 2.0 uF.



a) Calculate the average power delivered to the load when $R_o = 20 \text{ k}\Omega$ and $C_o = 1.1 \text{ uF}$.

b) Determine the settings of R_o and C_o that will result in the most average power being transferred to R_o .

c) What is the most average power in (b)? Is it greater then power in (a)?

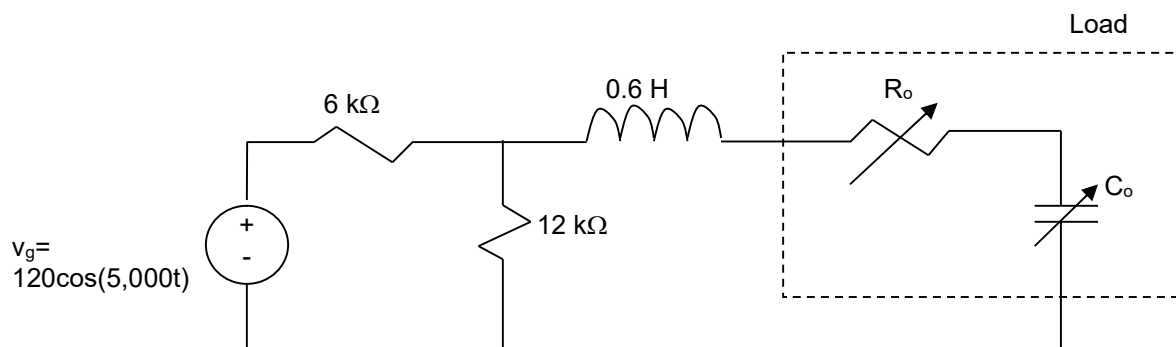
d) If there are no constraints on R_o and C_o , what is the maximum average power that can be delivered to a load?

e) What are the values of R_o and C_o for the condition of (d)?

f) Is the average power calculated in (d) larger than that calculated in (c)?

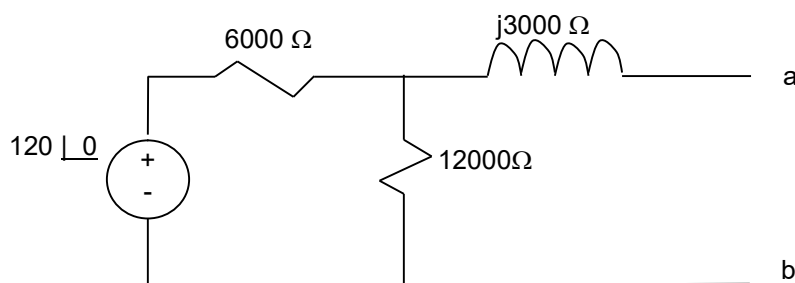
Solution:

9S. Assuming that R_o can be varied between 0 and 10 k Ω , determine the settings of R_o and C_o that will result in the most average power being transferred to load. Also, calculate the Maximum Average Power delivered to load.



Solution:

First find the circuit thevenin equivalent with respect to ports the load is connected to.
Step 1. Convert to Phasor Domain



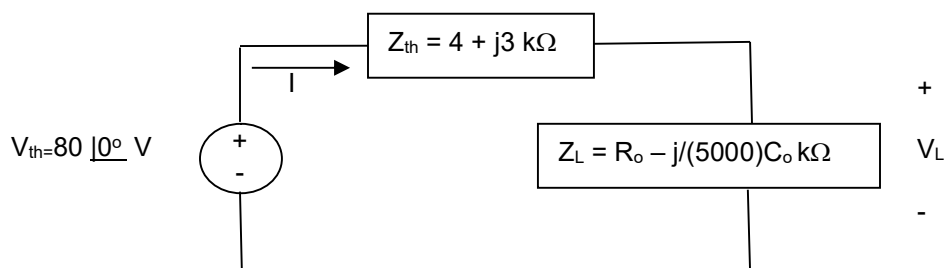
Step 2. Find Z_{th} by deactivating the source (short voltage source)

$$Z_{th} = Z_{ab \text{ eq.}} = (6000 \parallel 12000) + j3000 = 4000 + j3000 \Omega$$

Step 3. Find V_{th} (open circuit ab)

$$V_{th} = 120 * (12000/18000) = 80 \text{ V}$$

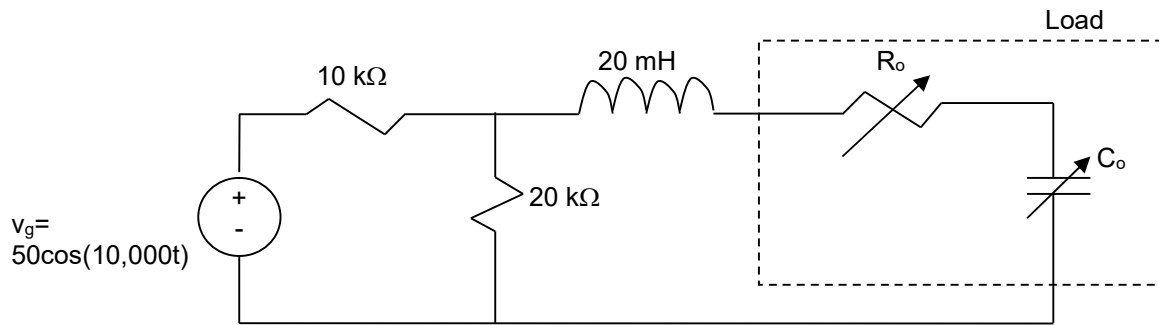
Resulting Thevenin equivalent is shown below:



The condition for maximum power transfer is $Z_{th} = Z_L^*$ therefore
 $4 + j3 = R_o + j/5C_o \rightarrow R_o = 4 \text{ k}\Omega$ & $1/(5000)C_o = 3 \text{ k}\Omega \rightarrow C_o = 66.67 \text{ nF}$

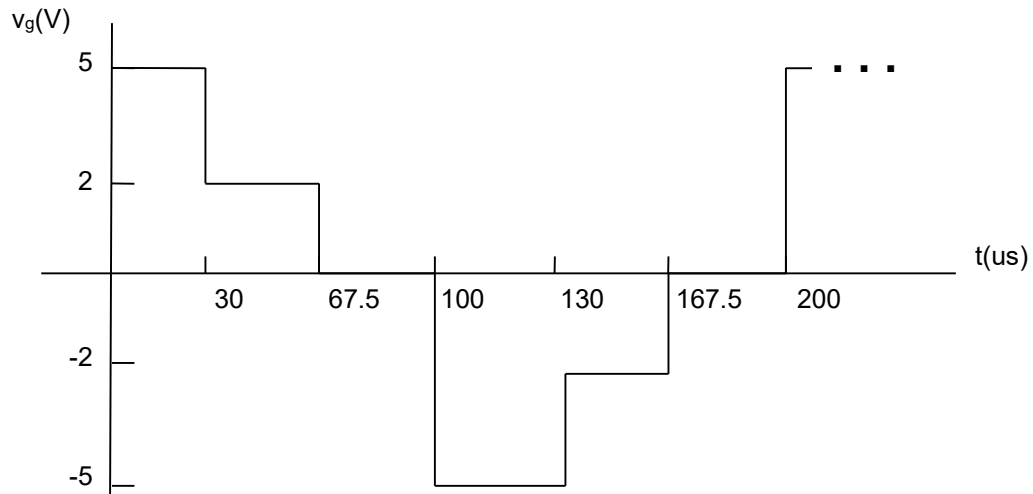
$$P_{max} = \text{Maximum Real Power} = |V_m|^2/8R_L = 80^2/(8*4000) = 0.2 \text{ W} = 200 \text{ mW}$$

9U. Assuming that R_o can only be varied between 0 and $30\text{ k}\Omega$, determine the settings of R_o and C_o that will result in the most average power being transferred to R_o . Also, calculate the Maximum Average Power delivered to load.



Solution:

10S. For the following Signal:



- Find the rms value of the periodic voltage.
- If the voltage is applied to the terminals of a 2.25 Ohm resistor, what is the average power dissipated by the resistor.

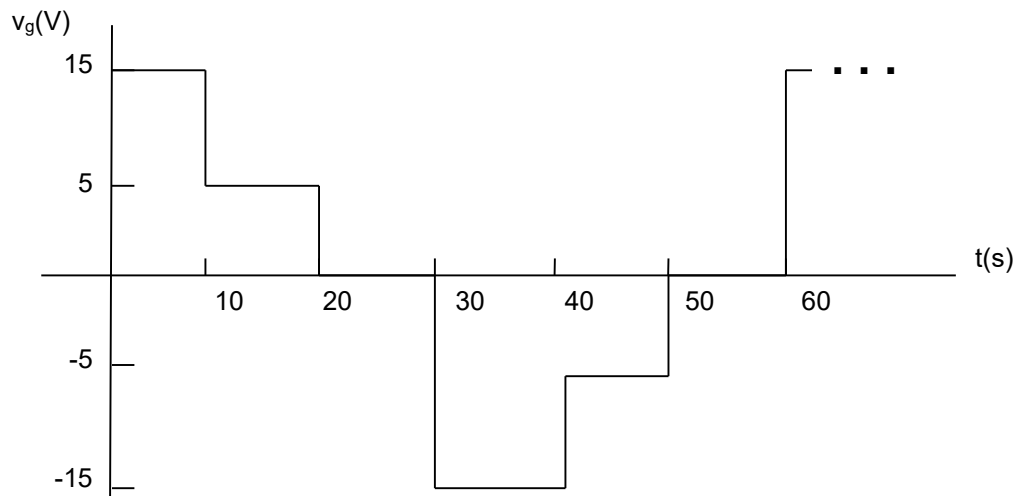
Solution:

a)

$$V_{rms} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} v^2(t) dt} = \sqrt{\frac{2}{200} \left\{ \int_0^{30} 5^2 dt + \int_{30}^{67.5} 2^2 dt + 0 \right\}} = \sqrt{\frac{2}{200} \{25(30 - 0) + 4(67.5 - 30) + 0\}} = 3\text{ V}$$

b) $P = V_{eff}^2/R = (3)^2/2.25 = 4\text{ W}$

10U. For the following Signal:



- Find the rms value of the periodic voltage.
- If the voltage is applied to the terminals of a 10 Ohm resistor, what is the average power dissipated by the resistor.

Solution:

11U. In your home you are simultaneously using a number of devices with different impedance profile resulting in the following power consumption profile:

Product	Power	Power Factor
Hair dryer	1200 W	
Television (tub-type)	110 VA	0.9 Lagging
Vacuum Cleaner	680 VAR	
Appliances	3000 VA	0.75 Lagging
Ecar Charging Station	4800 VA	0.85 Leading

- Determine the apparent power, power factor and total complex powers for the home.
- Do you need more capacitive or inductive load to improve your usage efficiency (power factor)?