Fundamentals of Electrical Circuits - Chapter 11

1S. What is the phase sequence of each of the following sets of voltages?

a) $v_a = 208\cos(wt + 27^o)$ V, $v_b = 208\cos(wt + 147^o)$ V, $v_c = 208\cos(wt - 93^o)$ V.

b) $v_a = 4160\cos(wt - 18^o)$ V, $v_b = 4160\cos(wt - 138^o)$ V, $v_c = 4160\cos(wt + 102^o)$ V.

Solution:

a) $\theta_b - \theta_a = +120^o$ and $\theta_c - \theta_a = -120^o$ → acb (negative) sequence

b) $\theta_b - \theta_a = -120^o$ and $\theta_c - \theta_a = +120^o$ → abc (positive) sequence

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1U. What is the phase sequence of each of the following sets of voltages?

a) $v_a = 250\cos(wt - 58^o)$ V, $v_b = 250\cos(wt - 178^o)$ V, $v_c = 250\cos(wt + 62^o)$ V.

b) $v_a = 110\cos(wt + 17^o)$ V, $v_b = 110\cos(wt + 137^o)$ V,, $v_c = 110\cos(wt - 103^o)$ V.

Solution:

2S. For each set of voltages, state whether or not the voltages from a balanced three-phase set. If the set is balanced, state whether the phase sequence is positive or negative. If the set is not balanced, explain why.

a) $v_a = 180\cos(377t)$ V, $v_b = 180\cos(377t - 120^o)$ V, $v_c = 180\cos(377t - 240^o)$ V.

b) $v_a = 180\sin(377t)$ V, $v_b = 180\sin(377t + 120^o)$ V, $v_c = 180\cos(377t - 120^o)$ V.

c) $v_a = -400\sin(377t)$ V, $v_b = 400\sin(377t + 210^o)$ V, $v_c = 400\cos(377t - 30^o)$ V.

d) $v_a = 200\cos(wt + 30^o)$ V, $v_b = 201\cos(wt + 150^o)$ V, $v_c = 200\cos(wt + 270^o)$ V.

e) $v_a = 208\cos(wt + 42^o)$ V, $v_b = 208\cos(wt - 78^o)$ V, $v_c = 208\cos(wt - 201^o)$ V.

f) $v_a = 240\cos(377t)$ V, $v_b = 240\cos(377t - 120^o)$ V, $v_c = 240\cos(397t + 120^o)$ V.

Solution:

Part	Same Amplitude	Same Freq.	120º ΔPhase	abc or acb	Conclusion
a	Yes	Yes	Yes	abc	Balanced, Positive (abc) Sequence
b	Yes	Yes	No	acb	Unbalanced phase
c	Yes	Yes	No	None	Unbalanced phase
d	No	Yes	Yes	acb	Unbalanced amplitude
e	Yes	Yes	No	None	Unbalanced phase
f	Yes	No	Yes	abc	Unbalanced frequency

2U. For each set of voltages, state whether or not the voltages from a balanced three-phase set. If the set is balanced, state whether the phase sequence is positive or negative. If the set is not balanced, explain why.

a) $v_a = 180\cos(29\pi t)$ V, $v_b = 180\cos(29\pi t + 120^o)$ V, $v_c = 180\cos(29\pi t - 120^o)$ V.

b) $v_a = 200\sin(377t)$ V, $v_b = (100*2)\sin(377t - 120^o)$ V, $v_c = 200\cos(377t + 120^o)$ V.

c) $v_a = -400\sin(377t + \pi/2)$ V, $v_b = 400\sin(377t - 150^o)$ V, $v_c = 400\cos(377t - 30^o)$ V.

d) $v_a = 200\cos(wt + 30^o)$ V, $v_b = 201\cos(wt + 150^o)$ V, $v_c = 200\cos(wt + 270^o)$ V.

Solution:

3S. The time-domain expressions for three line-to-neutral voltages at the terminals of a Y-Connected load are:

$v_{AN} = 169.71\cos(wt + 26^o)$ V

$v_{BN} = 169.71\cos(wt - 94^o)$ V

$v_{CN} = 169.71\cos(wt + 146^o)$ V

What are the time-domain expressions for the three line-to-line voltages v_{AB}, v_{BC} and V_{CA}?

Solution:

$v_{AN} = 169.71\ \underline{|26^o} \rightarrow v_{AB} = \sqrt{3}\, v_{AN}\ \underline{|30^o} = \sqrt{3}\ 169.71\ \underline{|56^o} = 293.95\ \underline{|56^o} \rightarrow V_{AB}(t)=293.95\cos(wt+56^o)$

$V_{BN} = 169.71\ \underline{|-94^o} \rightarrow v_{BC} = \sqrt{3}\, v_{BN}\ \underline{|30^o} = \sqrt{3}\ 169.71\ \underline{|-64^o} = 293.95\ \underline{|-64^o} \rightarrow V_{BC}(t)=293.95\cos(wt-64^o)$

$V_{CN} = 169.71\ \underline{|146^o} \rightarrow v_{CA} = \sqrt{3}\, v_{CN}\ \underline{|30^o} = \sqrt{3}\ 169.71\ \underline{|176^o} = 293.95\ \underline{|176^o} \rightarrow V_{CA}(t)=293.95\cos(wt+176^o)$

3U. The time-domain expressions for three line-to-neutral voltages at the terminals of a Y-Connected load are:

$v_{AN} = 125\cos(wt - 4^o)$ V
$v_{BN} = 125\cos(wt - 124^o)$ V
$v_{CN} = 125\cos(wt + 116^o)$ V

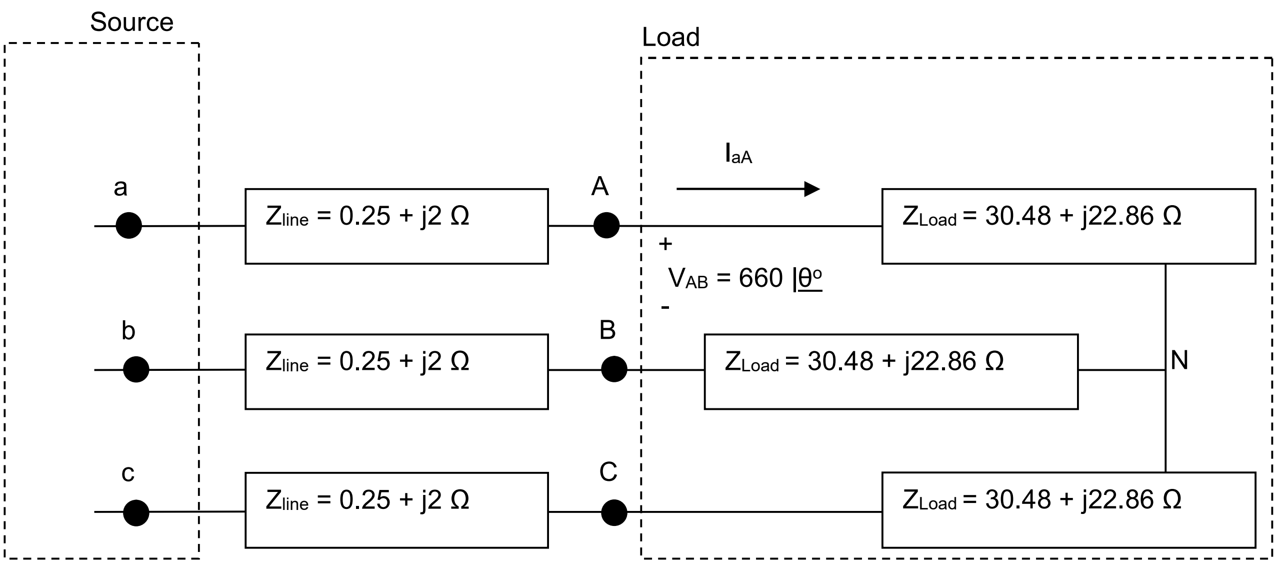
What are the time-domain expressions for the three line-to-line voltages v_{AB}, v_{BC} and V_{CA}?

Solution:

4S. The magnitude of the line voltage at the terminals of a balanced Y-connected load is 660V. The load impedance is 30.48 + j22.86 Ω/ϕ. The load is fed from a line that has an impedance of 0.25 + J2 Ω/ϕ.
a) What is the magnitude of the line current?
b) What is the magnitude of the line voltage at the source?

Solution:

a)



$$\mathbf{V_{AN}} = \frac{|V_{AB}|}{\sqrt{3}}\ |\underline{0-30^o} = \frac{660}{\sqrt{3}}\ |\underline{-30^o} = 381\ |\underline{-30^o}\ V$$

$$\mathbf{I_{aA}} = \frac{V_{AN}}{Z_{load}} = \frac{381\ |\underline{-30^o}}{30.48 + j22.86} = 3.92 - j9.20 = 10\ |\underline{-66.87^o}\ A$$

Therefore $|I_{aA}|$ =10 A

b)

Apply KVL to find $|V_{ab}|$ =

$$V_{ab} = V_{aA} + V_{AB} + V_{Bb} = Z_{line} I_{aA} + V_{AB} + Z_{line} I_{Bb} = Z_{line} I_{aA} + V_{AB} - Z_{line} I_{bB}$$

$$V_{ab} = (0.25 + j2)(10\underline{|-66.87^o}) + 660\underline{|0^o} - (0.25 + j2)(10\underline{|-186.87^o}) = 684.71\underline{|2.10^o}$$

$$Therefore \quad |Vab| = 684.71V$$

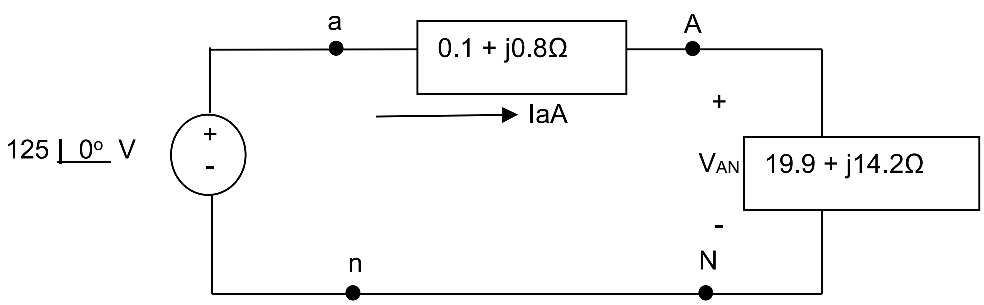
4U. The magnitude of the line voltage at the terminals of a balanced Y-connected load is 250V. The load impedance is 150 + j200Ω/ϕ. The load is fed from a line that has an impedance of 0.1 + J.5 Ω/ϕ.

a) What is the magnitude of the line current?
b) What is the magnitude of the line voltage at the source?

Solution:

5S. The magnitude of the phase voltage of an ideal balanced three phase Y-connected source is 125V. The source is connected to a balanced Y-connected load by a distribution line that has an impedance of 0.1+J0.8 Ω/ϕ. The load impedance is 19.9 + j14.2 Ω/ϕ. The phase sequence of the source is acb. Use the a-phase voltage of the source as the reference. Specify the magnitude and phase angle of the following quantities:

a) the three line currents.
b) the three line voltages at the source.
c) the three phase voltages at the load.
d) the three line voltages at the load.



Solution:

a) Find the a-phase line current from the above circuit

KVL → $IaA = \dfrac{125}{(0.1 + j0.8 + 19.9 + j14.2)} = \dfrac{125}{20 + j15} = 4 - j3 = 5\,|\underline{-36.87^o}A$

Negative (acb) Phase Sequence →

I_{bB} = 5 |-36.87° + 120° A = 5 |83.13° A

I_{cC} = 5 |-36.87° - 120° A = 5 |-156.87° A

b) Line Voltages at the source Vab, Vbc, Vca

$$Van = 125\,|\underline{0^o}\ V \quad Phase\ Voltage$$

$$Vab = Van(\sqrt{3}\,\underline{|-30^o}) = 216.51\underline{|-30^o}\ V$$

$$Vbc = 216.51\underline{|-30^o + 120^o} = 216.51\underline{|90^o}\ V$$

$$Vca = 216.51\underline{|-30^o - 120^o} = 216.51\underline{|-150^o}\ V$$

c) Phase Voltages at the load

$$V_{AN} = I_{aA} Z_L = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23\angle{-1.36^o}\ V \quad Phase\ Voltage$$

$$V_{BN} = 122.23\angle{-1.36^o + 120^o} = 122.23\angle{118.64^o}\ V$$

$$V_{CN} = 122.23\angle{-1.36^o - 120^o} = 122.23\angle{-121.36^o}\ V$$

d) Line Voltages at the load

Find the line voltage at a first & then using the fact that negative phase sequence to find b&c line voltages.

$$V_{AN} = 122.23\angle{-1.36^o}\ V \quad Phase\ Voltage$$

$$V_{AB} = V_{AN}(\sqrt{3}\angle{-30^o}) = 211.71\angle{-31.36^o}\ V$$

$$V_{BC} = 211.71\angle{-31.36^o + 120^o} = 211.71\angle{88.69^o}\ V$$

$$V_{CA} = 211.71\angle{-31.36^o - 120^o} = 211.71\angle{-151.36^o}\ V$$

5U. The magnitude of the phase voltage of an ideal balanced three phase Y-connected source is 250 kV. The source is connected to a balanced Y-connected load by a distribution line that has an impedance of 0.5+J2.2 Ω/ϕ. The load impedance is 19.5 + j7.8 Ω/ϕ. The phase sequence of the source is abc. Use the a-phase voltage of the source as the reference. Specify the magnitude and phase angle of the following quantities:

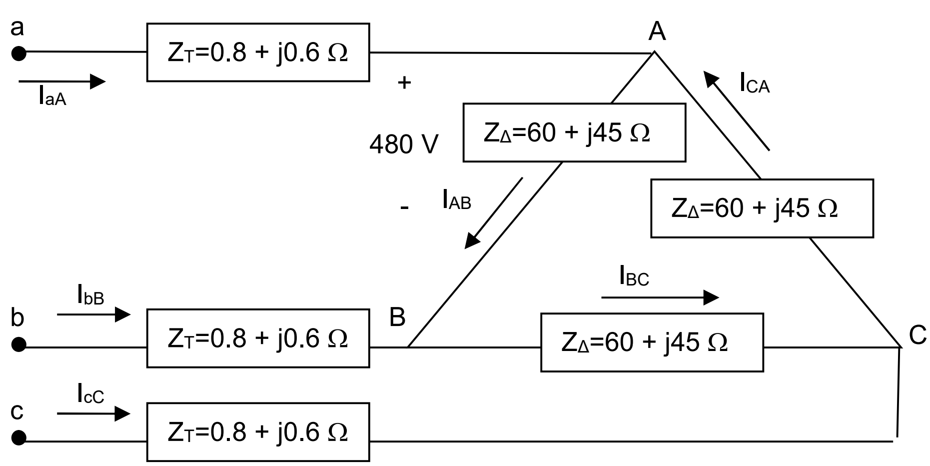
a) the three line currents.
b) the three line voltages at the source.
c) the three phase voltages at the load.
d) the three line voltages at the load.

Solution:

6S. A balanced Δ-connected load has an impedance of 60 + j45 Ω/ϕ. The load is fed through a line having an impedance of 0.8 + j0.6Ω/ϕ. The phase voltage at the terminals of the load is 480 V. The phase sequence is positive. Use V_{AB} as the reference.

a) Calculate the three phase currents of the load.
b) Calculate the three line currents.
c) Calculate the three line voltages at the sending end of the line.

Solution:



a) Phase Currents

$$I_{AB} = \frac{480}{60 + j45} = 6.4\,\underline{|-36.87^o}\,A$$

abc sequence

$$I_{BC} = 6.4\,\underline{|-120 - 36.87^o} = 6.4\,\underline{|-156.87^o}\,A$$

$$I_{CA} = 6.4\,\underline{|+120 - 36.87^o} = 6.4\,\underline{|+83.13^o}\,A$$

b) Line Currents

$$I_{aA} = \sqrt{3}\,\underline{|-30^o}\,I_{AB} = 11.09\,\underline{|-66.87^o}\,A$$

abc sequence

$$I_{bB} = 11.09\,\underline{|-120 - 66.87^o} = 11.09\,\underline{|-186.87^o}\,A$$

$$I_{cC} = 11.09\,\underline{|+120 - 66.87^o} = 11.09\,\underline{|53.13^o}\,A$$

c) Line voltage from Sending side

$$V_{ab} = \left[11.09\,\underline{|-66.87^o}\right]\left[0.8 + j0.6\right] + 480 - \left[11.09\,\underline{|-186.87^o}\right]\left[0.8 + j0.6\right] = 499.20\,\underline{|0^o}\ V$$

abc sequence

$$V_{bc} == 499.20\,\underline{|-120^o}\ V$$

$$V_{ca} == 499.20\,\underline{|120^o}\ V$$

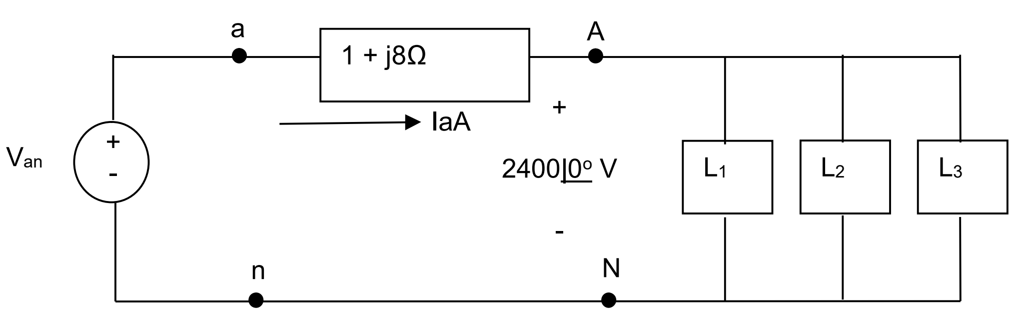
6U. A balanced Δ-connected load has an impedance of 100 + j20 Ω/ϕ. The load is fed through a line having an impedance of 0.5 + j2.5Ω/ϕ. The phase voltage at the terminals of the load is 660 V. The phase sequence is negative. Use V_{AB} as the reference.

a) Calculate the three phase currents of the load.
b) Calculate the three line currents.
c) Calculate the three line voltages at the sending end of the line.

Solution:

7S. A balanced three-phase distribution line has an impedance of 1+j8 Ω/ϕ. This line is used to supply three balanced three-phase loads that are connected in parallel. The Complex power at the three loads are L_1 =120 KVA at 0.96 pf lead, L_2 = 180 kVA at 0.80 pf lag and L_3 = 100.8 kW and 15.6 kVAR (magnetizing). The magnitude of the line voltage at the terminals of the loads is $2400\sqrt{3}\ V$.

a) What is the magnitude of the line voltage at the sending end of the line?
b) What is the percent efficiency of the distribution line with respect to average power?



Solution:

a)

$|V_{AB}| = \sqrt{3}\,|V_{AN}| \;\rightarrow\; 2400\sqrt{3} = \sqrt{3}\,|V_{AN}| \;\rightarrow\; |V_{AN}| = 2400$ Assume a is reference so phase is 0

Load 1, PF=0.96 & lead so $\theta<0 \rightarrow \theta=-\cos^{-1}(0.96) = -16.3^o$
$S_{1/\phi}$ =120,000cos(-16.3°)+ j120,000sin(-16.3°)=115,200 – j33,600 VA

Load 2, PF=0.80 & lag so $\theta>0 \rightarrow \theta=\cos^{-1}(0.80) = 36.9^o$
$S_{2/\phi}$ =180,000cos(36.9°)+ j180,000sin(36.9°)=143,943 + j108,076 VA

Load 3
$S_{3/\phi}$ =100,800 + j15,600 VA

Total Complex Power
$S_{T/\phi} = S_{1/\phi} + S_{2/\phi} + S_{3/\phi}$ = 360,000 + j90,000 VA

$S_{T/\phi} = (I_{aA}^*)(V_{AN})$

$$I_{aA}^* = \frac{360{,}000 + j90{,}000}{2400} = 150 + j37.5A$$

$$I_{aA} = 150 - j37.5A$$

$V_{an} = (1+j8)(I_{aA}) + 2400$ = 2850 + j1162.5= 3077.97 $\angle 22.19^o$ V

$|V_{ab}| = \sqrt{3}\,|V_{an}| = 5331.2$ V

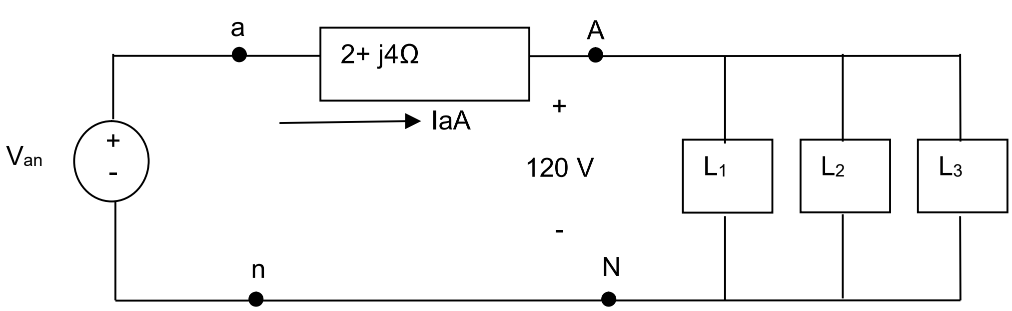
b)

$S_{g/\phi} = (I_{aA}^*)(V_{an})$ = (150-+j37.5)(2850+j1162.5)=383,900 + j281,250 VA

$$\%efficiency = \frac{Load\_Real\_Power}{Source\_Real\_Power} = \frac{360{,}000}{383{,}900} = 94\%$$

7U. A balanced three-phase distribution line has an impedance of 2+j4 Ω/ϕ. This line is used to supply three balanced three-phase Y-connected loads that are connected in parallel. The Complex power at the three loads are L_1 =200 KVA at 0.5 pf lag, L_2 = 80 kVA at 0.3 pf lead and L_3 = 300 kW and 100 kVAR (magnetizing). The magnitude of the phase voltage at the load terminals is 120 V.

a) What is the magnitude of the line voltage at the sending end of the line?
b) What is the percent efficiency of the distribution line with respect to average power?



Solution:

8S. The three pieces of computer equipment described below are installed as part of a computation center.

* DISK: 4.864 kW at 0.79 pf lag
* ZIP DRIVE: 17.636 kVA at 0.96 pf lag
* CPU: line current 73.8 A, 13.853 kVAR

Each piece of equipment is balanced three-phase load rated at 208 V line voltage. Calculate

a) the magnitude of the line current supplying these three devices
b) the power factor of the combined load.

Solution:

a)

Disk Load PF=0.79 & lag so $\theta>0 \rightarrow \theta=\cos^{-1}(0.79) = 37.8^o$
S_d =4864+ j4864(tan37.8°)=4864 + j3773 VA

Zip Load, PF=0.96 & lag so $\theta>0 \rightarrow \theta=\cos^{-1}(0.96) = 16.3^o$
S_z =17,636cos(16.3 °) + j17,636sin(16.3 °) = 16931 + j 4938

CPU Load,
Reactive Power = Q_c = $\sqrt{3}V_L I_L \sin(\theta)$
$13{,}853 = \sqrt{3}(208)(73.8)\sin(\theta) \rightarrow \sin\theta = 0.52$ & $\cos\theta = 0.85$ & $\tan\theta = 0.52/0.85$
S_c =13853(1 / tanθ) + j13,853 = 22,644 + j13,853

$S_T = S_d + S_z + S_c$ = 44439 + j22564 VA

$S_{T/\phi}$ = (1/3)S_T = 14813 + j7521 VA Magnitude of phase power

$S_{T/\phi} = V_{AN} I_{aA}{}^* = (208/\sqrt{3}) I_{aA}{}^* \rightarrow I_{aA}{}^* = 123 + j63 \rightarrow I_{aA} = 123 - j63 = 138 \angle{-27^o}$ A (rms)

b) Power Factor of the combined load
$\theta = \tan^{-1}(22564/44439) = 27^o$
$\theta > 0 \rightarrow$ lagging
PF = cos (27) = 0.89 lagging

8U. The three major componenets of an Electric car are listed below with their power profile:

* Motor: 800 W at 0.4 pf lag
* Instrument Panel: 50 VA at 0.8 pf lag
* Controllers, Wiring and connetions: effective phase current of 2 A, 10 VAR

Each component group is feed by balanced three-phase source rated at 72 V effective phase voltage. Determine the following:

a) magnitude of the line current supplying these three devices
b) power factor for the combined load.

Solution: