ENGR 253 LAB #5 - Fourier Series Analysis

Objective

Exploring the use Fourier Series Analysis and Synthesis methods for periodic signals.

Resources

- Course Lecture Material
- MATLAB or GNU Octave Development Environment

Background

1) Fourier Series Representation of Periodic Signals Analysis and Synthesis

   - Continuous-time

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t} \quad \text{Synthesis Equation}$$

$$a_k = \frac{1}{T}\int_T e^{-jkw_0 t} x(t)dt = \frac{1}{T}\int_T e^{-jk(2\pi/T)t} x(t)dt \quad \text{Analysis Equation}$$

   - Discrete-time

$$x[n] = \sum_{k=<N>} a_k e^{jkw_o n} = \sum_{k=<N>} a_k e^{jk(2\pi/N)n} \quad \mathit{for}\ k = m, m+1, \ldots, m+N-1 \quad \mathit{Fourier\ Series\ Synthesis\ Eq.}$$

$$a_k = \frac{1}{N}\sum_{n=<N>} x[n] e^{-jkw_o n} = \frac{1}{N}\sum_{n=<N>} x[n] e^{-jk(2\pi/N)n} \quad \mathit{Fourier\ Series\ Analysis\ Eq.}$$

MATLAB functions, fft() & iff(), implement Synthesis and Analysis equations. For the signal x[n] with the fundamental period N, two of the Discrete-Time Fourier Series (DTFS) related MATLAB are shown below:

```
a = (1/N) * fft(x)        % DTFS coefficients ak for 0 ≤ k ≤ N-1
x = N * ifft(a)           % x[n] for 0 ≤ k ≤ N-1
```

2) Sample Sound Data Files
   You can download bird chirp (11025 sample/second) and Handel Messiah (44100 sample/second) sound files from the following locations:

   * https://www.engrcs.com/components/chirp.wav
   * https://www.engrcs.com/components/handel.flac

3) Playing Sound Data Files
   Below is a code example for playing sound data file. In this case, a bird chirping.

```
info = audioinfo("chirp.wav")   % returns information about the content of sound file
y = audioread("chirp.wav");     % load the chirp.wav sound data file into variable y (n-by-2 matrix)
sound(y,11025);                 % output the value of y as an sound value at 11025 samples/second
stem(y);                        % graphs value stored in y
```

Note: If you copy the code, you may need to renter double quotes.

Experiment #1

Load the file Handel.flac and graph the file. Explain the graph horizontal axis, vertical axis and how does the graph relates to the physical data stored in the file?

Experiment #2

Synthesize a periodic discrete-time signal with period N=5 and the following DTFS coefficients

$$a_0 = 2, \quad a_2 = a_{-2}{}^* = e^{j\pi/4}, \quad a_4 = a_{-4}{}^* = e^{j\pi/3}$$

a) Based on the DTFS coefficients, do you expect x[n] to have complex-value, purely real, or purely imaginary? Explain your answer?

b) Using the DTFS coefficients given above, determine the values of a_0 through a_4.

c) Using MATLAB Synthesis function and values of a_k found in the pervious section to determine and graph the value of x[n] for $0 \le n \le 25$.

d) Determine and graph the value of x[n] using the synthesis equation directly {do not use ifft()}. What is the percent difference in x[n] energy between the direct method used here and MATLAB function used in part (c).

e) Graph x[n] magnitude, phase, real part and imaginary part using stem. Was your statement in part a correct?

Report Requirements

Lab and reports must be completed individually. All reports must be computer printed (Formulas and Diagrams may be hand drawn) and at minimum include:

For each Experiment

a) A clear problem statement; specifying items given and to be found.
b) Theory or process used.
c) Resulting circuits, calculation, tables, timing diagram, schematic and other relevant results.

For the report as a whole

a) Cover sheet with your name, class, lab, completion date and team members' names.
b) Lessons Learned from the experiments.
c) A new experiment and expected results which provide additional opportunity to practice the concepts in this lab.