ENGR 253 LAB #6 - Discrete Fourier Transform (DFT) and Inverse DFT

Objective

Utilizing Fourier Transform to analyze the original, the transformed and the approximated signals.

Resources

- Course Lecture Material
- MATLAB or GNU Octave Development Environment

Background

1) Fourier Transform for aperiodic and periodic signals

- Continuous-time

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}\,dt \xrightarrow{Approximateion\ \tau=dt\to 0} \lim_{\tau\to 0} \sum_{n=-\infty}^{\infty} x(n\tau)e^{-jw_0 n\tau}\tau$$

For a signal that is non-zero only in the range 0 < t ≤ T and T=Nτ where N is an integer, X=τ*fft(x) may be used to approximate X(jw). Each element will be calculated using the following:

$$X(k+1) = \tau \sum_{n=-\infty}^{N-1} x(n\tau)e^{-jw_k n\tau} \quad for \quad 0 \le k \le N$$

$$X(k+1) \approx X(jw_k) \quad for \quad w_k = \frac{2\pi k}{N\tau} \quad for \quad 0 \le k \le \frac{N}{2}$$

$$= \frac{2\pi k}{N\tau} - \frac{2\pi}{\tau} \quad for \quad \frac{N}{2} + 1 \le k \le N$$

N is assumed to be even and fft returns positive frequency sample before the negative frequency samples. Function fftshift() rearrange the samples such that they are placed in the vector from the most negative to most positive frequency.

iff() function performs the inverse fourier transform function.

- Discrete-time

$$X(e^{jw}) = \sum_{n=<N>} x[n]e^{-jwn} \quad Fourier\ Transform$$

$$x[n] = \frac{1}{2\pi}.\int_{2\pi} X(e^{jw})e^{jwn}\,dw \quad Fourier\ Inverse\ Transform$$

MATLAB functions, fft() & ifft() are designed to implement the above transformation equations. For a value of N that is less than number of samples in x or X, the functions may be used as shown below:

```
X = fft(x, N)        % N-point Fourier Transform of x, padded with zeros if X has less
                     %   than N points and truncated if it has more
x =  ifft(X,N)       % N-point inverse Fourier Transform of X,  returns the n-point
                     %   inverse DFT of vector X.
```

Note: For any x, ifft(fft(x)) approximates x within round off errors

2) Approximation Error (E)
Typically approximation error, goodness of an approximation, is determined by the size of energy difference between the approximated signal and the original signal. For this experiment use the difference in total energy over the nonzero period as measure of approximation error:

$$E = \sum_{k=<N>} x[n]^2 - \sum_{k=<N_1>} \overline{x[n]}^2 \quad where$$

$$x[n] = orignial\ signal\ (N\ terms)$$

$$\overline{x[n]} = approximated\ signal (N_1\ terms)$$

Experiment #1
Load sound file handel.flac, play the content of Handel as sound with 44,100 samples/second and graph the data. Use only the first 200,000 samples for this experiment (1 ≤ n <200,000). Show the correlation of what you hear with graph of the data.

Hint: Refer to lab 5 for accessing sound file instructions.

Experiment #2
Load the Handel file into y and run the following commands:

a) sound (y, 88200)
b) sound (y, 44100)
c) sound (y, 22050)

For each command explain what you hear, how long it played and the reason for the difference between a&b and b&c.

Experiment #3
Use fft() function to calculate and graph the Fourier Transform of the signal represented by sample values stored in the Handel file. You can limit your data to first 100,000 samples (1 ≤ n <100,000). Show the correlation of what you heard in Experiment 1 with graph of the data from this Experiment.

Experiment #4
Use ifft() function to approximate the original signal (recovered signal) from the Fourier Transform derived in Experiment #3 and graph the recovered signal.

Experiment #5
Calculate the error (E) between the original and recovered signals. Can you hear the difference between original signal and the recovered signal.

Experiment #6
Use MATLAB to produce sounds in order to determine your hearing range. Explain your process and your results.

Hint: A healthy young person who has not been exposed to excessively loud sounds and disease typically hears sounds from 20 to 20 kHz.

Report Requirements

Lab and reports must be completed individually. All reports must be computer printed (Formulas and Diagrams may be hand drawn) and at minimum include:

For each Experiment

a) A clear problem statement; specifying items given and to be found.
b) Theory or process used.
c) Resulting circuits, calculation, tables, timing diagram, schematic and other relevant results.

For the report as a whole

a) Cover sheet with your name, class, lab, completion date and team members' names.
b) Lessons Learned from the experiments.
c) A new experiment and expected results which provide additional opportunity to practice the concepts in this lab.