Signals & Systems - Chapter 4

1S. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $e^{-2(t-1)}u(t-1)$ **b)** $e^{-2|t-1|}$

Sketch and label the magnitude of each Fourier transform.

Solution: For Fourier Transform we have

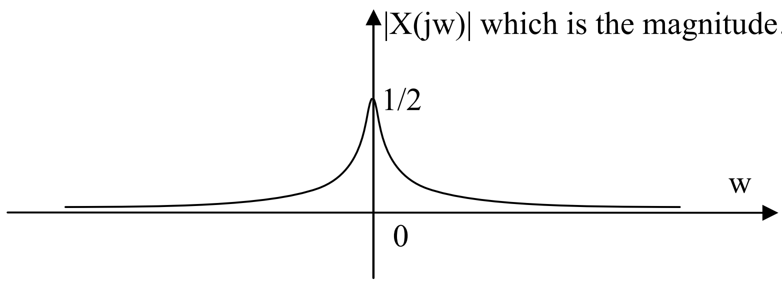
$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw$ *Fourier Transform Synthesis equation*

$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$ *Fourier Transform Analysis equation*

a) Using the Fourier Transform Analysis equation

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt = \int_{-\infty}^{\infty} e^{-2(t-1)}u(t-1)e^{-jwt}dt = \int_{1}^{\infty} e^{-2(t-1)}e^{-jwt}dt$$

$$X(jw) = \int_{1}^{\infty} e^{-(2+jw)t+2}dt = \frac{-1}{2+jw}e^{-(2+jw)t+2}\Big|_{1}^{\infty} = \frac{-1}{2+jw}(0 - e^{-(2+jw)+2}) = \frac{e^{-jw}}{2+jw}$$



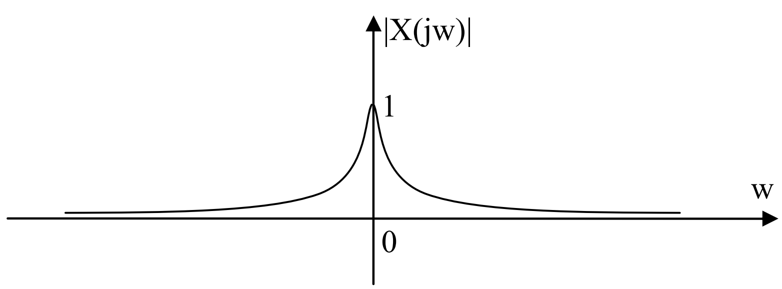
b) Using the Fourier Transform Analysis equation

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt = \int_{-\infty}^{\infty} e^{-2|t-1|}e^{-jwt}dt = \int_{1}^{\infty} e^{-2(t-1)}e^{-jwt}dt + \int_{-\infty}^{1} e^{2(t-1)}e^{-jwt}dt$$

$$X(jw) = \int_{1}^{\infty} e^{-(2+jw)t+2}dt + \int_{-\infty}^{1} e^{(2-jw)t-2}dt$$

$$\frac{-1}{2+jw}e^{-(2+jw)t+2}\Big|_{1}^{\infty} + \frac{1}{2-jw}e^{(2-jw)t-2}\Big|_{-\infty}^{1} = \frac{e^{-jw}}{2+jw} + \frac{e^{-jw}}{2-jw} =$$

$$X(jw) = \frac{4e^{-jw}}{4+w^2}$$



1U. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $e^{4(t-5)}u(-t+3)$ **b)** $e^{-4|t-3|}$

Sketch and label the magnitude of each Fourier transform.

Solution:

2S. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $\delta(t+1)+\delta(t-1)$ **b)** $\frac{d}{dt}\{u(-2-t)+u(t-2)\}$

Sketch and label the magnitude of each Fourier transform.

Solution: For Fourier Transform we have

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(jw)e^{jwt}dw \quad \textit{Fourier Transform Synthesis equation}$$

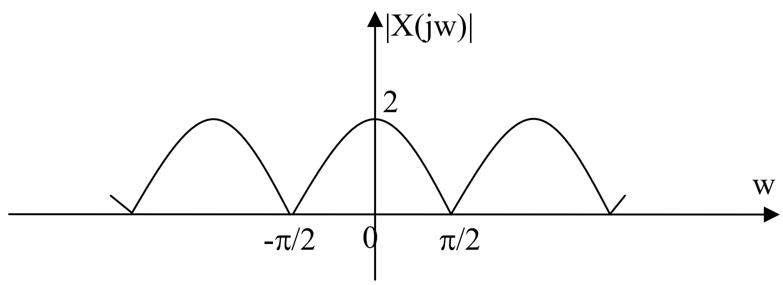
$$X(jw)=\int_{-\infty}^{\infty}x(t)e^{-jwt}dt \quad \textit{Fourier Transform Analysis equation}$$

a) Use the Fourier Transform Analysis Equation

$$X(jw)=\int_{-\infty}^{\infty}x(t)e^{-jwt}dt=\int_{-\infty}^{\infty}\{\delta(t+1)+\delta(t-1)\}e^{-jwt}dt$$

we know that when t=0 then δ(t) =1otherwise δ(t) =0 therefore

$$X(jw)=e^{jw}+e^{-jw}=2\cos w$$

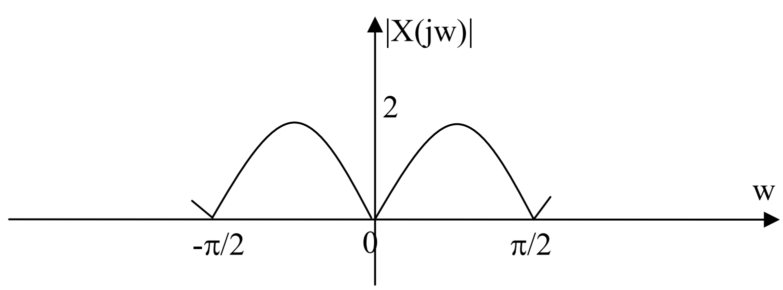


b) Use the Fourier Transform Analysis Equation

$$X(jw)=\int_{-\infty}^{\infty}x(t)e^{-jwt}dt=\int_{-\infty}^{\infty}\frac{d}{dt}\{u(-2-t)+u(t-2)\}e^{-jwt}dt$$

we know that $\frac{d\{u(t)\}}{dt}=\partial(t)$ therefore

$$X(jw)=\int_{-\infty}^{\infty}\{\delta(t-2)-\delta(t+2)\}e^{-jwt}dt=e^{-2jw}-e^{2jw}=-2j\sin(2w)$$



2U. Use the Fourier transform analysis equation to calculate the Fourier transform of:

a) $\delta(t+2)+\delta(-t+2)$ **b)** $\frac{d}{dt}\{u(-3-t)+u(t-3)\}$

Sketch and label the magnitude of each Fourier transform.

Solution:

3S. Determine the Fourier transform of each of the following periodic signals:

a) $\sin(2\pi t + \frac{\pi}{4})$ **b)** $1 + \cos(6\pi t + \frac{\pi}{8})$

Solution:

a) signal $x(t) = \sin(2\pi t + \frac{\pi}{4})$ is periodic with period (Fundamental Period, T=1 → $w_0 = 2\pi$)

The easiest way would be use Euler's Identity to find the Fourier Series coefficients.

$$x(t) = \sin(2\pi t + \frac{\pi}{4}) = \frac{1}{2j}(e^{j(2\pi t+\pi/4)} - e^{j(2\pi t+\pi/4)}) = \frac{1}{2j}e^{j\pi/4}e^{j2\pi t} - \frac{1}{2j}e^{-j\pi/4}e^{-j2\pi t}$$

Therefore $a_1 = \frac{1}{2j}e^{j\pi/4}$ $a_{-1} = -\frac{1}{2j}e^{-j\pi/4}$

We know this signal is periodic with period 1 and we know that a periodic signal in the above form has transfer function as shown below:

$$X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_0) = 2\pi a_{-1}\delta(w + w_0) + 2\pi a_1 \delta(w - w_0)$$

$$X(jw) = -\frac{\pi}{j}e^{-j\pi/4}\delta(w + 2\pi) + \frac{\pi}{j}e^{j\pi/4}\delta(w - 2\pi)$$

b) signal $x(t) = 1 + \cos(6\pi t + \frac{\pi}{8})$ is periodic with period (Fundamental Period, T=1/3 → $w_0 = 6\pi$)

The easiest way would be use Euler's Identity to find the Fourier Series coefficients.

$$x(t) = 1 + \cos(6\pi t + \frac{\pi}{8}) = 1 + \frac{1}{2}(e^{j(6\pi t+\pi/8)} + e^{-j(6\pi t+\pi/8)}) = 1 + \frac{1}{2}e^{j\pi/8}e^{j6\pi t} + \frac{1}{2}e^{-j\pi/8}e^{-j6\pi t}$$

Therefore $a_1 = \frac{1}{2}e^{j\pi/8}$ $a_{-1} = \frac{1}{2}e^{-j\pi/8}$ $a_0 = 1$

This signal is periodic:

$$w = \frac{2\pi}{N} = \frac{\pi}{8} \rightarrow \ Period\ N = 16$$

Fourier Series Coefficients can be used to to write the X(jw) as:

$$X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_0) = 2\pi a_{-1}\delta(w + w_0) + 2\pi a_0 \delta(w) + 2\pi a_1 \delta(w - w_0)$$

$$X(jw) = \pi e^{-j\pi/8}\delta(w + 6\pi) + 2\pi\delta(w) + \pi e^{j\pi/8}\delta(w - 6\pi)$$

3U. Determine the Fourier transform of each of the following periodic signals:

a) $\cos(2\pi t - \frac{\pi}{3})$ **b)** $10 + \sin(5\pi t + \frac{3\pi}{8})$

Solution:

4S. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of:

a) $X_1(jw) = 2\pi\delta(w) + \pi\delta(w-4\pi) + \pi\delta(w+4\pi)$

b) $X_2(jw) = \begin{cases} 2 & for \quad 0 \le w \le 2 \\ -2 & for \quad -2 \le w < 0 \\ 0 & for \quad |w| > 2 \end{cases}$

Solution:

a)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw \quad Fourier\ Transform\ Synthesis\ equation$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{2\pi\delta(w) + \pi\delta(w-4\pi) + \pi\delta(w+4\pi)\}e^{jwt}dw$$

$$x(t) = \frac{1}{2\pi}\{2\pi e^{0} + \pi e^{j4\pi t} + \pi e^{-j4\pi t}\} = 1 + \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t} = 1 + \cos(4\pi t)$$

b)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw \quad Fourier\ Transform\ Synthesis\ equation$$

$$x(t) = \frac{1}{2\pi}\int_{0}^{2} 2e^{jwt}dw + \frac{1}{2\pi}\int_{-2}^{0} -2e^{jwt}dw = \frac{1}{j\pi t}(e^{j2t} - 1\} - \frac{1}{j\pi t}(1 - e^{-j2t})$$

$$x(t) = \frac{1}{j\pi t}(e^{j2t} + e^{-j2t}) - \frac{2}{j\pi t}$$

4U. Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of:

a) $X_1(jw) = 3j\pi\delta(w-2\pi) + \pi\delta(w-4\pi) - 3j\pi\delta(w+2\pi) + \pi\delta(w+4\pi)$

b) $X_2(jw) = \begin{cases} 1 & for \quad 0 \le w \le 5 \\ -1 & for \quad -5 \le w < 0 \\ 0 & for \quad |w| > 5 \end{cases}$

Solution:

5S. Determine whether each of the following statements is true or false. Justify your answers.
a) An odd and imaginary signal always has an odd and imaginary Fourier transform.
b) The convolution of an odd Fourier transform with an even Fourier Transform is always odd.

Solution:

a) Form Fourier transform properties table we have
odd signal x(t) → purely imaginary and odd Fourier transform X(jw)
F{purely imaginary signal jx(t)} → jX(jw) using linearity

Based on the above two statements we can conclude that an odd and imaginary signal jx(t) always has an odd and real Fourier transform therefore the problem statement is **false**.

b) We have
1) odd Fourier transform corresponds to an odd signal
2) even Fourier transform corresponds to an even signal

Convolution of an even F.T. with an odd F.T. in time domain is multiplication of even and odd function which results in an odd function therefore the problem statement is **True**.

5U. Determine whether each of the following statements is true or false. Justify your answers.

a) A sin() function will always have pure imaginary even Fourier Transfer function.
b) A tan() function will always have pure imaginary odd Fourier Transfer function.
c) A cos() function will always have real and even Fourier Transfer function.

Solution:

6S. Compute the Fourier transform for each of the following Signals:

a) $[e^{-at}\cos w_o t]u(t) \quad for \quad a>0$

b) $e^{-3|t|}\sin 2t$

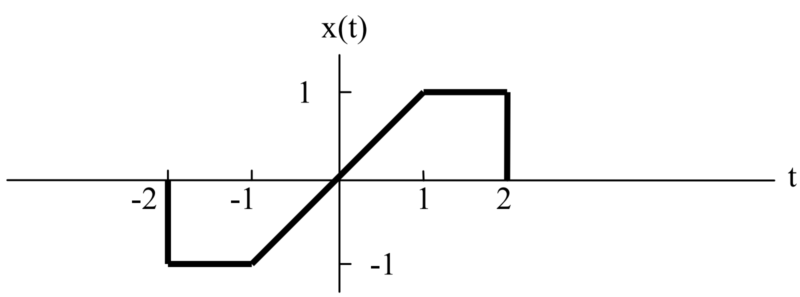
c) $x(t) = 1+\cos \pi t \quad for \quad |t| \le 1$
$\quad 0 \quad for \quad |t|>1$

d) $\sum_{k=0}^{\infty} a^k \delta(t-kT) \quad for \quad |a|<1$

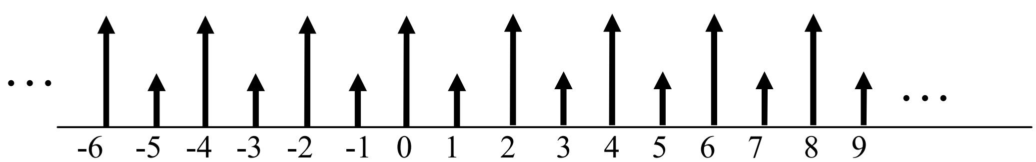
e) $[te^{-2t}\sin 4t]u(t)$

f) $\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$

g)



h)



i) $x(t) = 1-t^2 \quad for \quad 0<t<1$
$\quad 0 \quad Otherwise$

j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

Solution:

a)

$$x(t) = [e^{-at}\cos w_o t]u(t) \quad for \quad a>0$$

$$x(t) = \frac{1}{2}e^{-at}\{e^{jw_0 t} + je^{-jw_0 t}\}u(t) = \{\frac{1}{2}e^{-(a-jw_0)t} + \frac{1}{2}e^{-(a+jw_0)t}\}u(t)$$

From the Fourier Transform Table:

$$e^{-at}u(t) \xrightarrow{F} \frac{1}{a+jw}$$

Therefore→

$$X(jw) = \frac{1}{2(a - jw_0 + jw)} + \frac{1}{2(a + jw_0 + jw)}$$

b)

$$x(t) = e^{-3|t|}\sin(2t) = e^{-3t}\sin(2t)u(t) + e^{3t}\sin(2t)u(-t)$$

$$x(t) = \frac{1}{2j}e^{-3t}\{e^{j2t} - e^{-j2t}\}u(t) + \frac{1}{2j}e^{3t}\{e^{j2t} - e^{-j2t}\}u(-t)$$

$$x(t) = x_1(t) + x_2(t) \quad where$$

$$x_1(t) = \frac{1}{2j}e^{-(3-2j)t}u(t) - \frac{1}{2j}e^{-(3+2j)t}u(t)$$

$$x_2(t) = \frac{1}{2j}e^{-(-3-2j)t}u(-t) - \frac{1}{2j}e^{-(-3+2j)t}u(-t) = -x_1(-t) \Rightarrow= X_2(jw) = -X_1(-jw)$$

$$X_1(jw) = \frac{1}{2j(3 - j2 + jw)} - \frac{1}{2j(3 + j2 + jw)}$$

$$X(jw) = X_1(jw) + X_2(jw) = \frac{1}{2j(3 - j2 + jw)} - \frac{1}{2j(3 + j2 + jw)} - \frac{1}{2j(3 - j2 - jw)} + \frac{1}{2j(3 + j2 - jw)}$$

$$X(jw) = \frac{1}{9 + (w+2)^2} - \frac{1}{9 + (w-2)^2}$$

c)

$$x(t) = 1 + \cos \pi t \quad for \quad |t| \leq 1$$

$$0 \quad for \quad |t| > 1$$

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt = \int_{-1}^{1}(1 + co\pi t)e^{-jwt}dt = \int_{-1}^{1}(1 + \frac{1}{2}e^{+j\pi t} + \frac{1}{2}e^{-j\pi t})e^{-jwt}dt$$

$$X(jw) = \{\frac{e^{-jwt}}{-jw} + \frac{e^{j(\pi-w)t}}{2j(\pi-w)} + \frac{e^{j(\pi-w)t}}{2j(\pi-w)}\Bigg|_{-1}^{1} = \frac{2\sin w}{w} + \frac{\sin w}{\pi - w} - \frac{\sin w}{\pi + w}$$

d)

$$x(t) = \sum_{k=0}^{\infty} a^k \delta(t - kT) \quad for \quad |a| < 1$$

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt = \int_{-\infty}^{\infty}\sum_{k=0}^{\infty} a^k \delta(t - kT)e^{-jwt}dt$$

$$Note: \delta(t - kT) = 1 \; forvt = KT \; otherwise \; \delta(t - kT) = 0 \; therefore:$$

$$X(jw) = \sum_{k=0}^{\infty} a^k e^{-jwkT} = \sum_{k=0}^{\infty}(a^1 e^{-jwT})^k$$

$$Apply \;\; finite \; sum \Rightarrow \sum_{k=0}^{-\infty} a^k = \frac{1}{1-a} \; when \, |a| < 1$$

$$X(jw) = \frac{1}{1 - ae^{-jwT}}$$

e)

$$x(t) = [te^{-2t}\sin 4t]u(t) = \frac{t}{2j}[e^{-2t}e^{j4t} - e^{-2t}e^{-j4t}]u(t) = \frac{t}{2j}\left[e^{-(2-j4)t} - e^{-(2+j4)t}\right]u(t)$$

$$From\ table: te^{-at}u(t) \xrightarrow{F} \frac{1}{(a+jw)^2}$$

$$X(jw) = \frac{1}{j2}\left\{\frac{1}{(2-j4+jw)^2} - \frac{1}{(2+j4+jw)^2}\right\}$$

f)

$$x(t) = \left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$$

From the Fourier Transform Table:

$$x_1(t) = \frac{\sin \pi t}{\pi t} \xrightarrow{F} X_1(jw) = 1\ for\ |w| < \pi\ otherwise\ X_1(jw) = 0$$

$$apply\ the\ time\ shifting\ and\ time\ scaling\ properties \Rightarrow$$

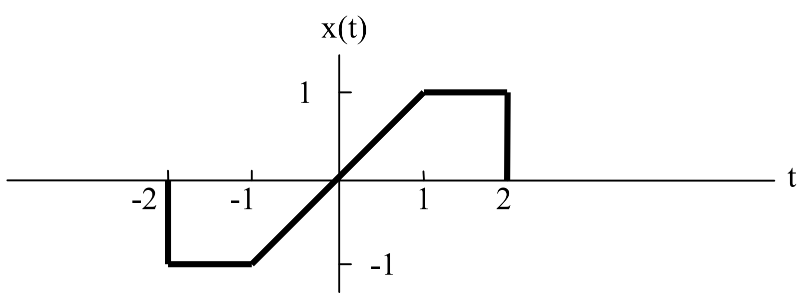
$$x_2(t) = \frac{\sin 2\pi(t-1)}{\pi(t-1)} \xrightarrow{F} X_2(jw) = e^{-j2w}\ for\ |w| < 2\pi\ otherwise\ X_1(jw) = 0$$

$x(t)=x_1(t)\ x_2(t)$ → $X(jw)=\{1/2\pi\}\{X_1(jw)*X_2(jw)\}$

$$X(jw) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_2(j\theta)X_1(j(w-\theta))d\theta$$

X(jw) =		
	e^{-jw}	$\|w\| < \pi$
	$(1/2\pi)\{3\pi+w\}e^{-jw}$	$-3\pi < w < -\pi$
	$(1/2\pi)\{3\pi+w\}e^{-jw}$	$\pi < w < 3\pi$
	0	otherwise

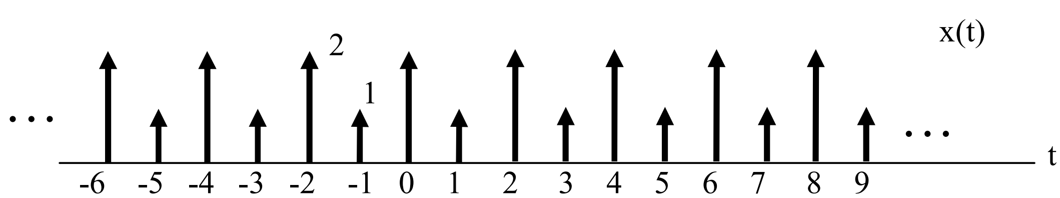
g)



$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt = \int_{-2}^{-1} -e^{-jwt}dt + \int_{-1}^{1} te^{-jwt}dt + \int_{1}^{2} e^{-jwt}dt$$

$$X(jw) = \frac{2j}{w}\left[\cos 2w - \frac{\sin w}{w}\right]$$

h)



$$if\ x_1(t)=\sum_{k=-\infty}^{\infty}\delta(t-2k)\ then\ x(t)=2x_1(t)+x_1(t-1)\ therefore$$

$$X(jw)=X_1(jw)[2+e^{jw}]=\pi\sum_{k=-\infty}^{\infty}\delta(w-\pi k)[2+e^{jw}]$$

i)

$$x(t)=1-t^2 \quad for \quad 0<t<1$$
$$0 \qquad Otherwise$$

$$X(jw)=\int_{-\infty}^{\infty}x(t)e^{-jwt}dt=\int_0^1(1-t^2)e^{-jwt}dt$$

$$X(jw)=\frac{1}{jw}+\frac{2e^{-jw}}{-w^2}-\frac{2e^{-jw}-2}{jw^2}$$

j) x(t)= $\sum_{n=-\infty}^{+\infty}e^{-|t-2n|}$ is periodic with period 2 therefore

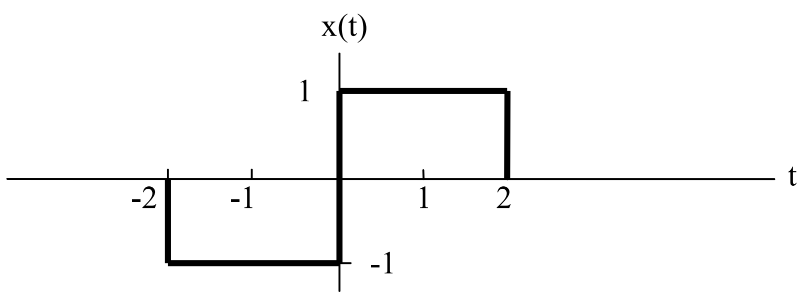
$$x(t)=\sum_{n=-\infty}^{+\infty}e^{-(t-2n)}u(t-2n)+\sum_{n=-\infty}^{+\infty}e^{(t-2n)}u(-t+2n)$$

$$X(jw)=\pi\sum_{k=-\infty}^{+\infty}\left[\frac{1}{1-e^{-2}}\left\{\frac{1-e^{-2(1+jw)}}{1+jw}-e^{-2}\frac{1-e^{2(1+jw)}}{1-jw}\right\}\right]\delta(w-k\pi)$$

6U. Compute the Fourier transform for each of the following Signals:

a) $e^{-4|t|}\cos 2t$

b)



c)
$$x(t)=4+3t^2 \quad for \quad 0<t<1$$
$$0 \qquad Otherwise$$

d) $\sum_{n=-\infty}^{+\infty}e^{-|2t-2n|}$

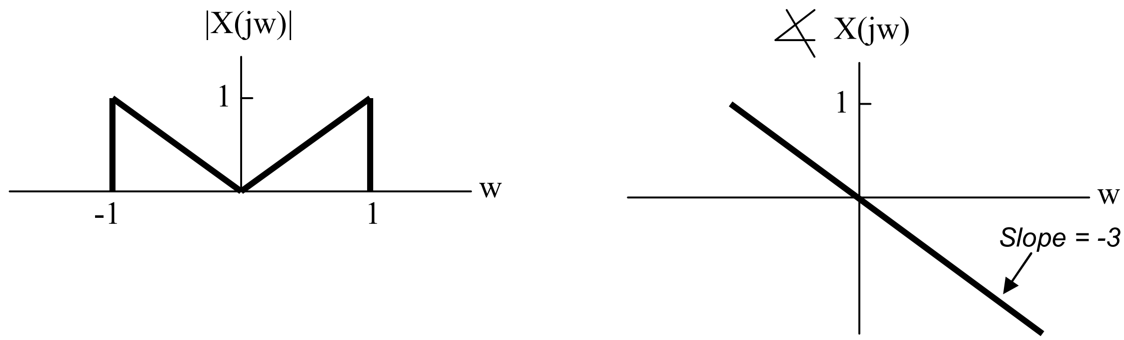
Solution:

7S. Determine the continuous-time signal corresponding to each of the following transforms.

a) $X(jw)=\dfrac{2\sin[3(w-2\pi)]}{(w-2\pi)}$

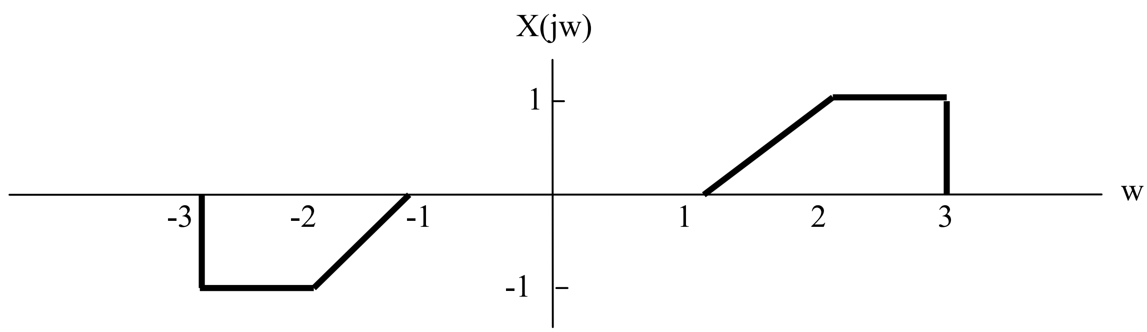
b) $X(jw) = \cos(4w + \pi/3)$

c) X(jw) as given by the following magnitude and phase plots



d) $X(jw) = 2[\delta(w-1) - \delta(w+1)] + 3[\delta(w-2\pi) + \delta(w+2\pi)]$

e) X(jw) as shown in the following figure



Solution:

a)

$$X(jw) = \frac{2\sin[3(w-2\pi)]}{(w-2\pi)}$$

Inverse Fourier Transform Table

$$X_1(jw) = \frac{2\sin wT_1}{w} \xrightarrow{F^{-1}\{\}} 1 \;\; for\; |t| < T_1 \;\; otherwise\; x_1(t) = 0$$

$$X_2(jw) = j(w - w_0)) \xrightarrow{F^{-1}\{\}} e^{jw_0 t} x_1(t)$$

therefore

$$x(t) = e^{j2\pi t} \qquad |t| < 3$$

$$0 \qquad Otherwise$$

b)

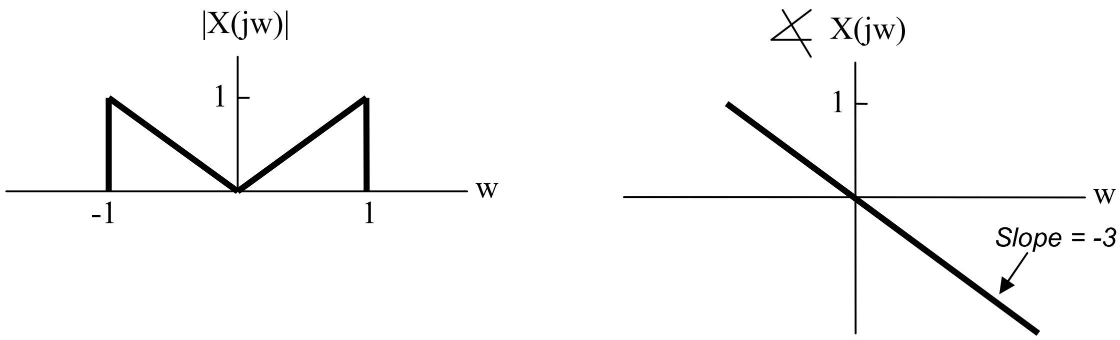
c) $X(jw) = \cos(4w + \pi/3) = \frac{1}{2}\{e^{j(4w+\pi/3)} + e^{-j(4w+\pi/3)}\} = \frac{1}{2}e^{j4w}e^{j\pi/3} + \frac{1}{2}e^{-j4w}e^{-j\ \ /3}$

Inverse Fourier Transform Table

$$e^{-jwt_0} \overset{F^{-1}\{\}}{\rightarrow} \delta(t - t_0)$$

Therefore

$$x(t) = \frac{1}{2}e^{j\pi/3}\delta(t+4) + \frac{1}{2}e^{-j\pi/3}\delta(t-4)$$



$$X(jw) = |X(jw)|\, e^{j\{phase\{X(jw)\}\}}$$

$$Inv.\ F.T. \Rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jw_0t}dw = \frac{1}{2\pi}\int_{-1}^{0} -we^{j(-3w)}e^{jw_0t}dw + \frac{1}{2\pi}\int_{0}^{1} we^{j(-3w)}e^{jw_0t}dw$$

$$x(t) = \frac{1}{\pi}\left[\frac{\sin(t-3)}{t-3} + \frac{\cos(t-3)-1}{(t-3)^2}\right]$$

d)

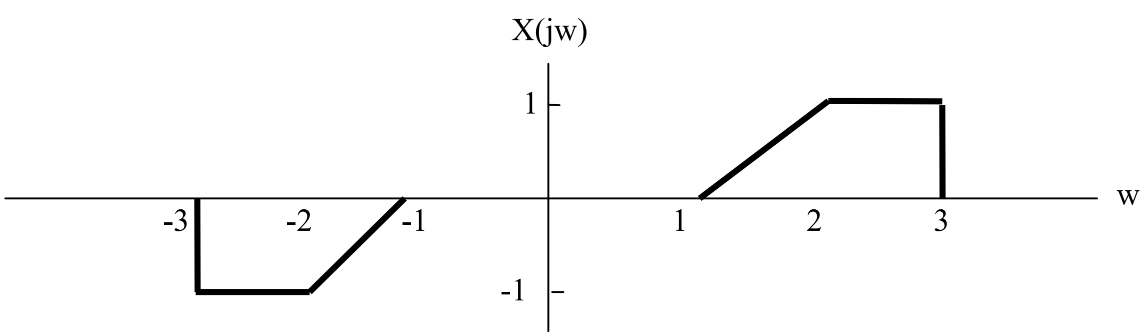
$$X(jw) = 2[\delta(w-1) - \delta(w+1)] + 3[\delta(w-2\pi) + \delta(w+2\pi)]$$

$$Inv..\ F.T.\ Table \Rightarrow$$

$$2\pi\delta(w-w_0) \overset{Inv.FT\{\}}{\rightarrow} e^{jw_0t}$$

$$x(t) = \frac{2}{2\pi}[e^{jt} - e^{-jt}] + \frac{3}{2\pi}[e^{j2\pi t} + e^{j2\pi t}] = \frac{2j}{\pi}\mathrm{sin}t + \frac{3}{\pi}\cos(2\pi t)$$

e)



$$Inv.\ F.T. \Rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jw_0t}dw$$

$$x(t) = \frac{1}{2\pi}\int_{-3}^{-2} -e^{jw_0t}dw + \frac{1}{2\pi}\int_{-2}^{-1}(1+t)e^{jw_0t}dw + \frac{1}{2\pi}\int_{1}^{2}(-1+t)e^{jw_0t}dw + \frac{1}{2\pi}\int_{2}^{3} e^{jw_0t}dw$$

$$x(t) = \frac{\cos 3t}{j\pi t} + \frac{\sin t - \sin 2t}{j\pi t^2}$$

7U. Determine the continuous-time signal corresponding to each of the following transforms.

a) $X(jw) = \dfrac{3\sin[5(w-5\pi)]}{(w-5\pi)}$

b) $X(jw) = \sin(2w - 2\pi/3)$

c) $X(jw) = 5[\delta(-w-3) - \delta(-w+3)] + 9[\delta(w-21\pi) + \delta(w+21\pi)]$

Solution:

8S. Compute Fourier transform for the following signal:

$$x(t) = \sum_{k=0}^{\infty} a^k \delta(t-kT) \quad for \quad |a| < 1$$

Solution:

$$x(t) = \sum_{k=0}^{\infty} a^k \delta(t-kT) \quad for \quad |a| < 1$$

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} a^k \delta(t-kT) e^{-jwt} dt$$

$$Note: \delta(t-kT) = 1 \; for \; t = KT \; otherwise \; \delta(t-kT) = 0 \; therefore:$$

$$X(jw) = \sum_{k=0}^{\infty} a^k e^{-jwkT} = \sum_{k=0}^{\infty} (a^1 e^{-jwT})^k$$

$$Apply \;\; finite \; sum \Rightarrow \sum_{k=0}^{-\infty} a^k = \frac{1}{1-a} \; when \; |a| < 1$$

$$X(jw) = \frac{1}{1 - ae^{-jwT}}$$

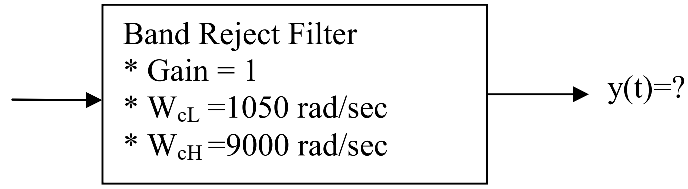
8U. Compute Fourier transform for the following signals:

$$x(t) = 5\sum_{k=0}^{\infty} (\frac{a}{2})^k \delta(t-kT) \quad for \quad |a| < 2$$

Solution:

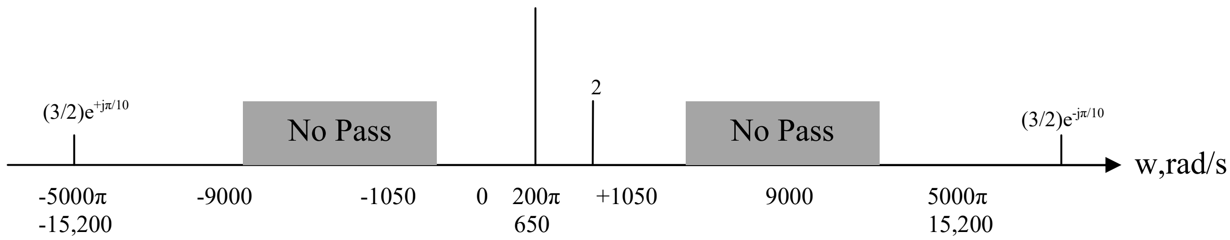
9S. Signal x(t) is input to a filter as shown in the following diagram:

x(t)= $6 + 2e^{j200\pi t} + 3\cos(5000\pi t - \pi/10)$



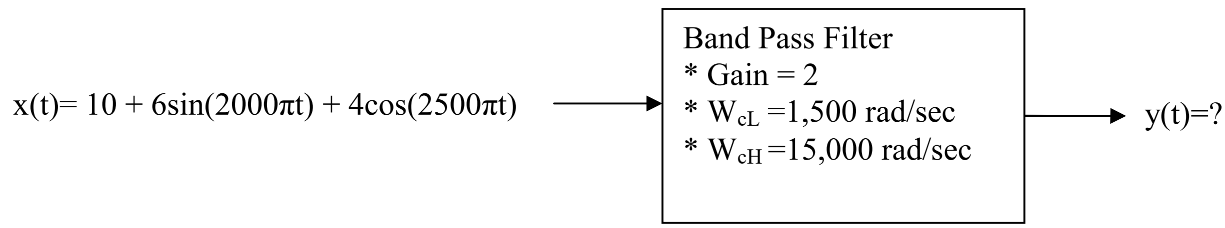
Derive the expression for the output y(t) in time domain.

Solution:



$$x(t) = 6 + 2e^{j200\pi t} + 3\cos(5000\pi t - \pi/10)$$

9U. Signal x(t) is input to a filter as shown in the following diagram:



Derive the expression for the output y(t) in time domain.

Solution:

10S. Consider the signal

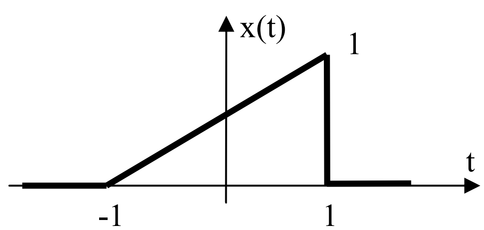
$$x(t) = \begin{cases} 0 & \text{for } |t| > 1 \\ (t+1)/2 & \text{for } -1 \le t \le 1 \end{cases}$$

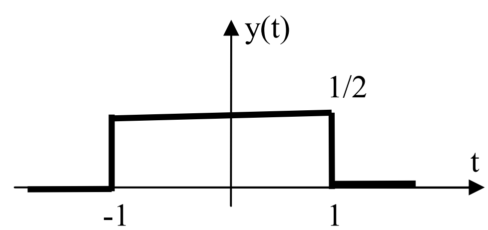
a) **With the help of Fourier Transform pair table, determine the closed-form expression for X(jw)**
b) **Take the real part of your answer to part (a), and verify that it is the Fourier transform of the even part of x(t).**
c) **What is the Fourier transform of the odd part of x(t)?**

Solution:

a)

in order to be able to use the tables, we need to make the following conversions:





We can rewrite x(t) as $x(t) = \frac{1}{2}\int_{-\infty}^{t} y(t)dt - u(t-1)$ and then use the following transformations from the table:

$$\int_{-\infty}^{t} y(t)dt \xrightarrow{F} \frac{1}{jw}Y(jw) + \pi Y(0)\delta(w)$$

$$Square\ wave: y(t) = 1/2\ for\ |t| < 1\ and\ y(t) = 0\ for\ |t| > 1 \xrightarrow{F} \frac{\sin w}{w}$$

$$z(t-1) \xrightarrow{F} e^{-jw}Z(jw)$$

$$u(t) \xrightarrow{F} \frac{1}{jw} + \pi\delta(w)$$

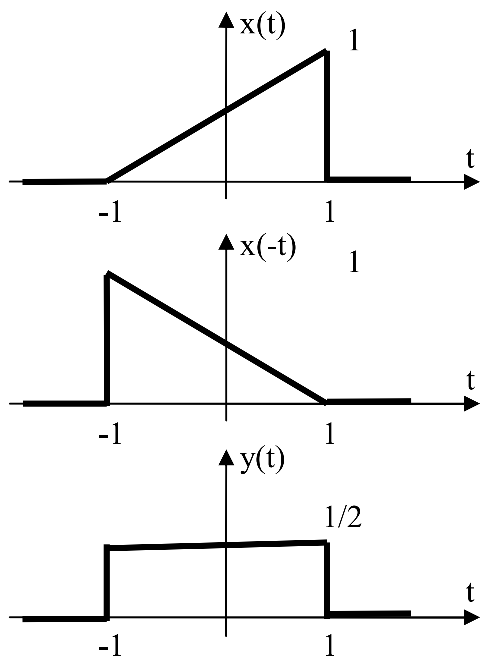
Which leads to:

$$X(jw) = \frac{1}{jw}\{\frac{\sin w}{w}) - e^{-jw}\{\frac{1}{jw} + \pi\delta(w)\}$$

$$X(jw) = \frac{\sin w}{jw^2} - \frac{e^{-jw}}{jw} - \pi e^{-jw}\delta(w)$$

b)

Even part {x(t)} = {x(t) + x(-t)} / 2



$$Square\ wave: y(t) = 1/2\ for\ |t| < 1\ and\ y(t) = 0\ for\ |t| > 1 \xrightarrow{F} \frac{\sin w}{w}$$

Therefore F{even part of x(t)}=sin(w) / w

c)

x(t) = {odd part of x(t)} + {even part of x(t)}
{odd part of x(t)}=x(t) - {even part of x(t)}

$$F\{odd\ part\ of\ x(t)\} = \frac{\sin w}{jw^2} - \frac{e^{-jw}}{jw} - \frac{\sin w}{w} = \frac{\sin w}{jw^2} - \frac{\cos w}{jw}$$

10U. Consider the signal

$$x(t) = \begin{cases} 0 & \text{for} \quad |t| > 2 \\ t/2 & \text{for} \quad -2 \le t \le 2 \end{cases}$$

a) **With the help of Fourier Transform pair table, determine the closed-form expression for X(jw)**

b) **Take the real part of your answer to part (a), and verify that it is the Fourier transform of the even part of x(t).**

c) **What is the Fourier transform of the odd part of x(t)?**

Solution: