Signals & Systems - Chapter 7

1S. x(t) is a signal with X(jw) = 0 when |w| > w_M. y(t) is another signal with the Fourier transform Y(jw) = 2X(j(w – w_c)). Determine a signal m(t) such that y(t) = x(t)m(t)

Solution:

Fourier Transform Property Table → F^{-1}{2X(j(w-wc))} = 2 $e^{jw_c t} x(t)$

Therefore m(t)= **2** $e^{jw_c t}$

1U. x(t) is a signal with X(jw) = 0 when |w| > w_M. y(t) is another signal with the Fourier transform Y(jw) = 2X(j(w – w_c)) + 2X(j(w + w_c)). Determine a signal m(t) such that y(t) = x(t)m(t)

Solution:

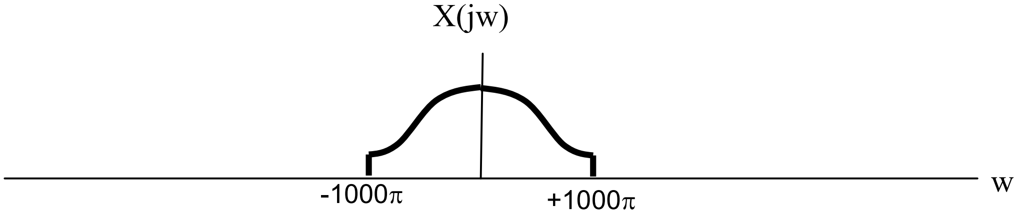
2S. x(t) is a real-signal with X(jw)=0 when |w| > 1,000π. Answer the following questions when y(t) = $e^{jw_c t} x(t)$ **:**

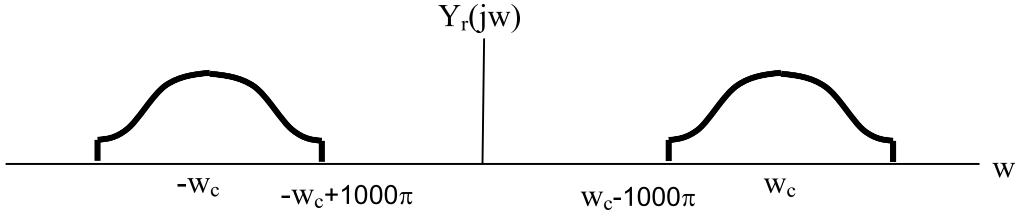
a) What (if any) limitation on w_c is needed to ensure that x(t) is recoverable from y(t)?
b) What (if any) limitation on w_c is needed to ensure that x(t) is recoverable from Real{y(t)}?

Solution:

a) Y(jw)=F{y(t)}= F{ $e^{jw_c t} x(t)$ } = X(j(w-w_c))
Since y(t) is simply a shifted version of x(t), it can be recovered by simply shifting y(t) back. Therefore there is no constraint on the w_c.

b) $y_r(t)$ = Real{y(t)} = Real {x(t)(cos(w_ct) + jsin(w_ct))} = x(t)cos(w_ct)
F{$y_r(t)$} = F{x(t)cos(w_ct)} = ½ [X(j(w-w_c) + X(j(w+w_c)]





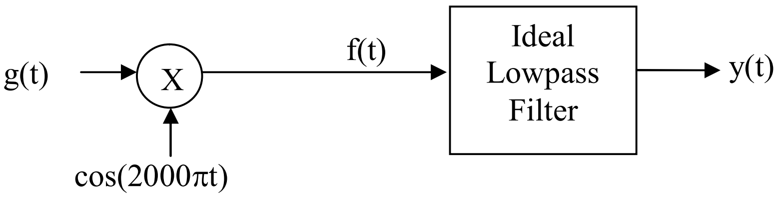
As long as the two parts are not overlapping, x(t) is recoverable → |w_c|>1000π

2U. x(t) is a real-signal with X(jw)=0 when |w| > 3,000π. Answer the following questions when y(t) = $\cos(w_c t)x(t)$ **:**

a) What (if any) limitation on w_c is needed to ensure that x(t) is recoverable from y(t)?
b) What (if any) limitation on w_c is needed to ensure that x(t) is recoverable from Real{y(t)}?

Solution:

3S. x(t) is a real signal where X(jw) = 0 for |w| > 2,000π. Amplitude modulated x(t) is g(t) = x(t) sin(2,000πt). g(t) is further modulated as shown in the following Figure where g(t) is the input, y(t) is the output. The ideal low pass filter has cutoff frequency 2,000π and passband gain of 2. Determine the value of output, y(t).



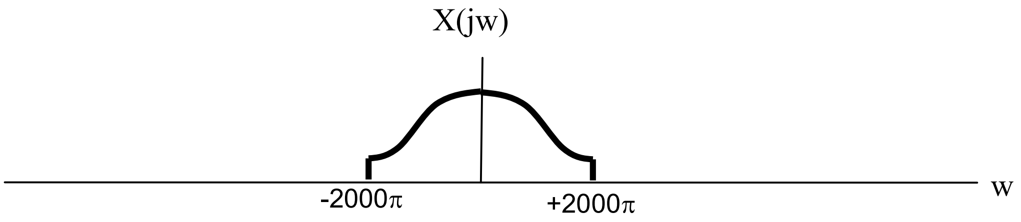
Solution:

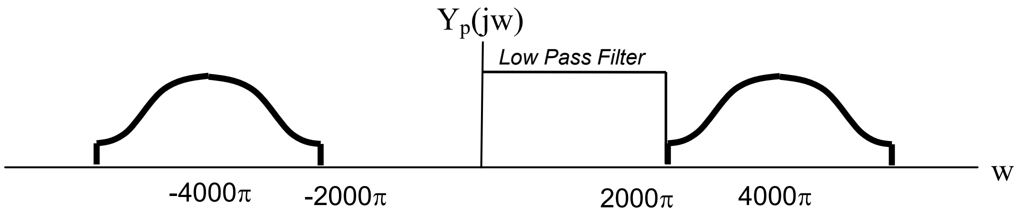
$g(t) = x(t)\sin(2000\pi t)$
take Fourier transform
$G(jw) = \boldsymbol{F}\{g(t)\} = 1/2j\, X(j(w-2000\pi)) - 1/2j\, X(j(w+2000\pi))$

$(f(t) = g(t).\cos(2000\pi t)$
$F(jw) = \boldsymbol{F}\{f(t)\} = 1/2\, G(j(w-2000\pi)) + 1/2\, G(j(w+2000\pi))$

Make substitution
$F(jw) =1/4jX(j(w-2000\pi-2000\pi)) - 1/4j\, X(j(w+2000\pi-2000\pi)) + 1/4j\, X(j(w-2000\pi+2000\pi)) - 1/4j\, X(j(w+2000\pi+2000\pi))$

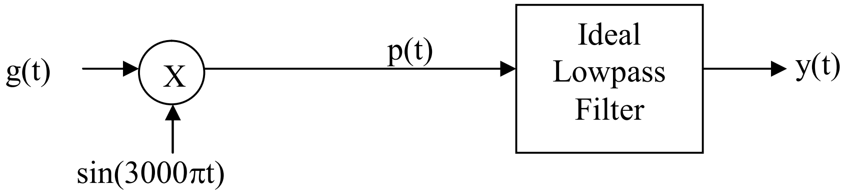
$F(jw) =1/4j\, X(j(w-4000\pi)) - 1/4j\, X(j(w+4000\pi))$





All of the signal falls outside the pass band of the low pass filter therefore y(t) = 0.

3U. x(t) is a real signal where X(jw) = 0 for |w| > 1,000π. Amplitude modulated x(t) is g(t) = x(t) cos(3,000πt). g(t) is further modulated as shown in the following Figure where g(t) is the input, y(t) is the output. The ideal low pass filter has cutoff frequency 1,500π and pass band gain of 2. Determine the value of output, y(t).



Solution:

4S. $y(t)=g(t)(\sin(400\pi t))$ is passed through a ideal low pass filter with cutoff frequency of 400π and pass band gain of 2. Determine the signal at the output of the low pass filter where:

$x(t) = \sin(200\pi t) + 2\sin(400\pi t)$
$g(t) = x(t)\sin(400\pi t)$

Solution:

$g(t) = x(t)\sin(400\pi t)$
take Fourier transform

$$G(jw) = \boldsymbol{F}\{g(t)\} = 1/2j\ X(j(w-400\pi)) - 1/2j\ X(j(w+400\pi))$$

$y(t) = g(t).\sin(400\pi t)$

$$Y(jw) = \boldsymbol{F}\{y(t)\} = 1/2j\ G(j(w-400\pi)) - 1/2j\ G(j(w+400\pi))$$

Make substitution

$$Y(jw) = -\ 1/4\ X(j(w-800\pi) + 1/4\ X(jw) - 1/4\ X(jw) + 1/4\ X(j(w+800\pi))$$
$$Y(jw) = -\ 1/4\ X(j(w-800\pi) + 1/4\ X(j(w+800\pi))$$

The only portion of y(t) that goes through the filter is ¼ $\sin(400\pi t)$ which has a frequency less than or equal 400π
→ Output is ½ $\sin(400\pi t)$

4U. $x(t) = \{\cos(800\pi t) + \sin(600\pi t)\}(\sin(600\pi t))$ is passed through a ideal low pass filter with cutoff frequency of 800π and pass band gain of 8. Determine the signal at the output of the low pass filter.

Solution:

5S. The signal $x(t) = \dfrac{\sin(1{,}000\pi t)}{\pi t}$ is transmitted using a modulator to create the signal, w(t) = (x(t) + A) $\cos(10{,}000\pi t)$. Determine the largest permissible value of the modulation index, m, such that asynchronous demodulation can be used to recover x(t) from w(t).

Solution:

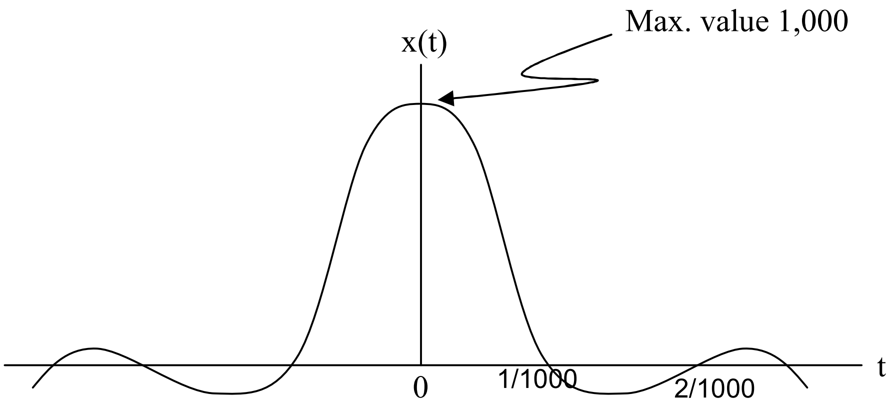
The two conditions required for this type of demodulation are:

1) Carrier frequency wc is much higher than the maximum signal frequency, w_M
   Carrier Frequency, $w_c = 10000\pi$
   Max Signal Carrier Frequency, $w_M = 1000\pi$
   → this condition is met since $w_c >> w_M$

2) Envelop value of the modulated signal must be positive
   In order to meet this condition value of "A" must be such that

   $x(t) + A > 0$ for all values of t

X(t) is a sinc function as shown below:



Min. value of x(t) occurs at t_{min} = (1/1000 + 2/1000)/2 = 3/2000

$x_{min} = x(t_{min}) = \dfrac{\sin(1{,}000\pi(3/2000))}{\pi(3/2000)} = -\dfrac{2000}{3\pi}$

Therefore Min. possible value of A must be equal to $\dfrac{2000}{3\pi}$ so that the Envelop remain positive

→ Modulation Index, m = $\frac{x(t)\ Maximum}{Min.\ Possible\ Value\ of\ A} = -\frac{1000}{2000/3\pi} = \frac{3\pi}{2} = 4.7$

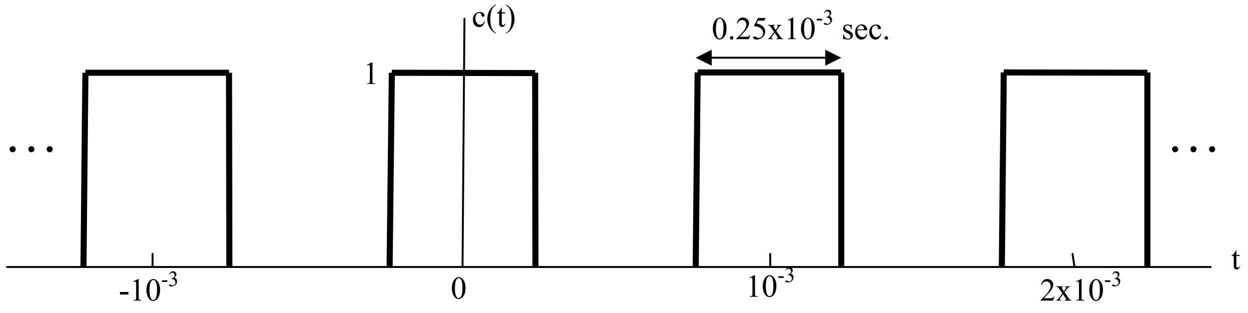
5U. The signal $x(t) = \dfrac{\sin(5{,}000\pi t)}{\pi t}$ **is transmitted using a modulator to create the signal,**

w(t) = (x(t) + A) cos(50,000πt).

Determine the largest permissible value of the modulation index, m, such that asynchronous demodulation can be used to recover x(t) from w(t).

Solution:

6S. x(t) is multiplied by c(t) (rectangular pulse train) shown below:



a) What are the constraint on X(jw) such that x(t) is recoverable from the product y(t) = x(t)c(t) by using an ideal low pass filter?
b) If conditions from part (a) are true, determine the low pass filter cutoff frequency w_c, and the passband gain, A in order to recover x(t) from y(t)=x(t)c(t).

Solution:

a)

Similar to impulse train sampling, the requirements are that $2\pi/T_s > 2w_M$

where:

w_M is maximum frequency of x(t)

$T_s = 10^{-3}$ is the sampling period which is

The constrain on X(jw) are derived from $2\pi/10^{-3} > 2W_M \rightarrow W_M < 1000\pi$

X(jw) = 0 where $|w| > w_M$

b)

Y(jw) = F{y(t)=x(t)c(t)} = $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)C(jw-\theta)d\theta$ From Fourier properties table

Y(jw) = $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\left[\sum_{k=-\infty}^{+\infty}\frac{2\sin kw_0T_1}{k}\delta(w-kw_0)\right]d\theta$ From Fourier Transform table

where $W_o = 2\pi/10^{-3} = 2000\pi \rightarrow T_1 = (0.25\text{x}10^{-3})/2$

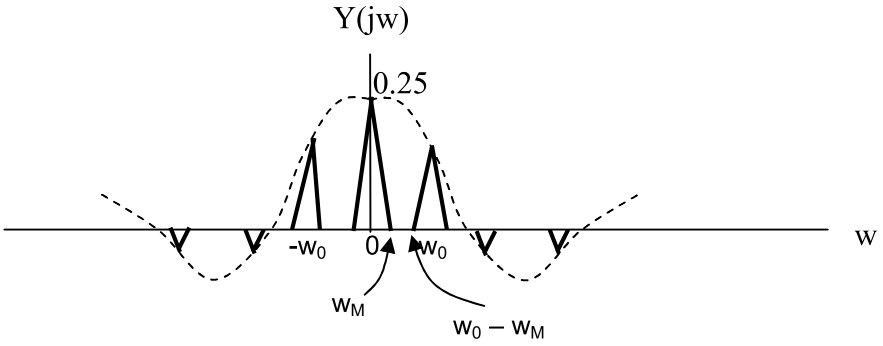
We are only interested in signal where w=0 → k=0

$$\lim_{k-->0}\left[\frac{2\sin(kw_oT_1)}{k}\right] \quad \underset{\Rightarrow}{Apply\ Hopital\ Rule} \quad \left[\frac{2w_oT_1\cos(kw_oT_1)}{1}\right] = 2w_oT_1$$

Y(jw) = $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)[2w_oT_1\delta(w-\theta)]d\theta$

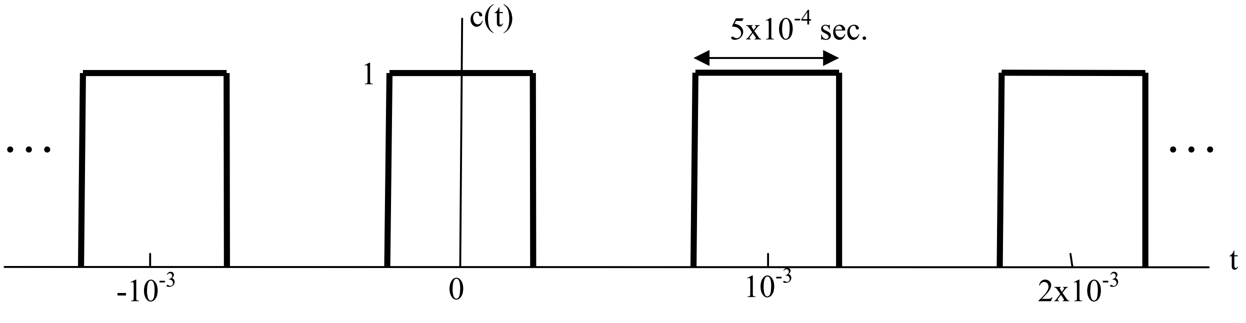
Apply impulse respond sifting rule and use T1 = $0.25\text{x}10^{-3}/2$, Wo=2,000π

Y(jw) = $\frac{1}{2\pi}X(jw)[2w_oT_1] = \frac{X(jw)}{4} \quad \Rightarrow \quad X(jw) = 4Y(jw)$



To recover x(t) the low pass filter should have cut off frequency needs to be between w_M and $(w_0 - w_M)$. And the gain A=1/0.25 = 4.

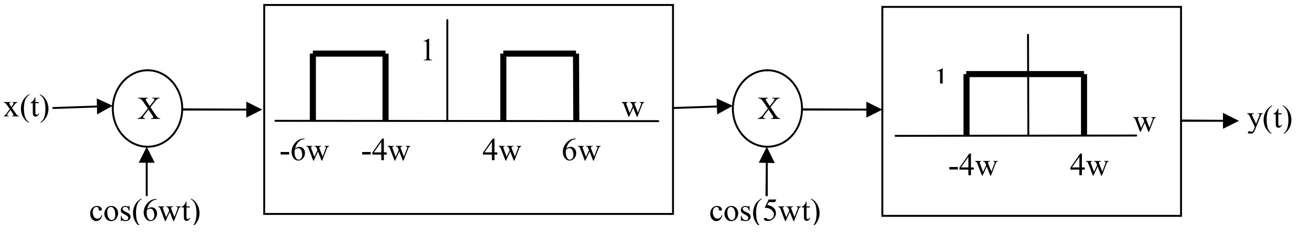
6u. x(t) is multiplied by c(t) (rectangular pulse train) shown below:

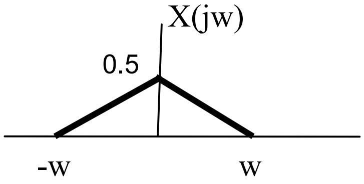


a) What are the constraint on X(jw) such that x(t) is recoverable from the product y(t) = x(t)c(t) by using an ideal low pass filter?
b) If conditions from part (a) are true, determine the low pass filter cutoff frequency w_c, and the passband gain, A in order to recover x(t) from y(t)=x(t)c(t).

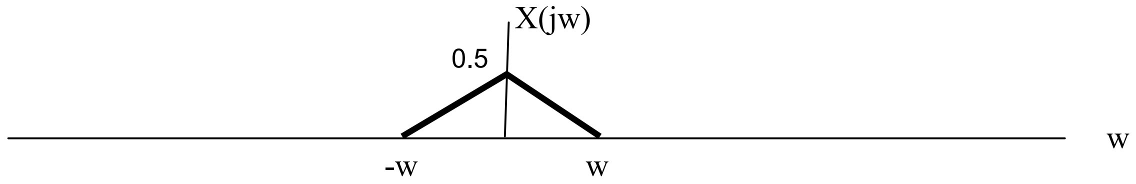
Solution:

7S. Determine and sketch Y(jw), the frequency spectrum of y(t) for the following system. Where X(jw) is the Fourier Transform of x(t):

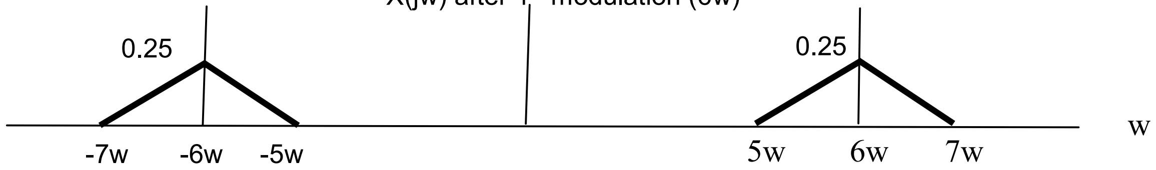




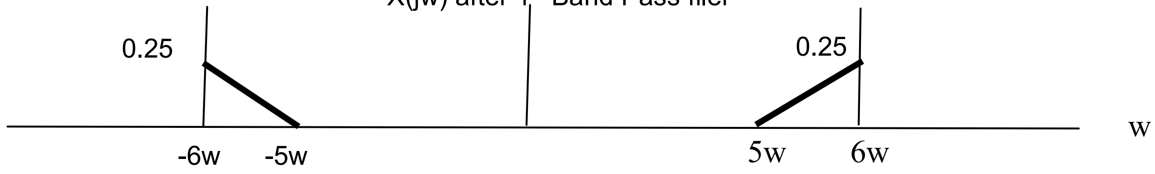
Solution:



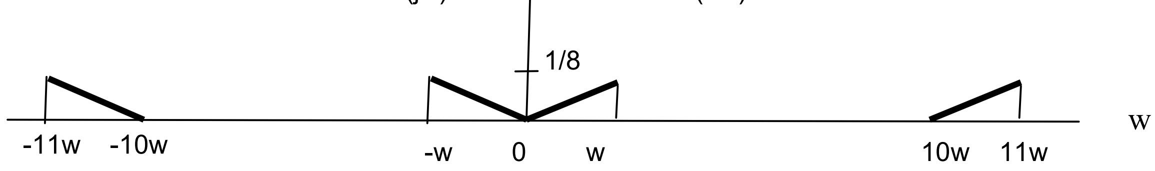
X(jw) after 1st modulation (6w)



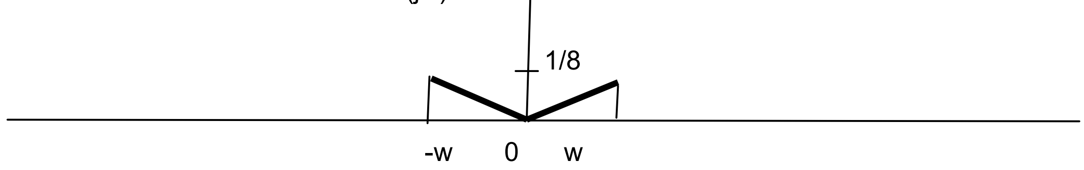
X(jw) after 1st Band Pass filer



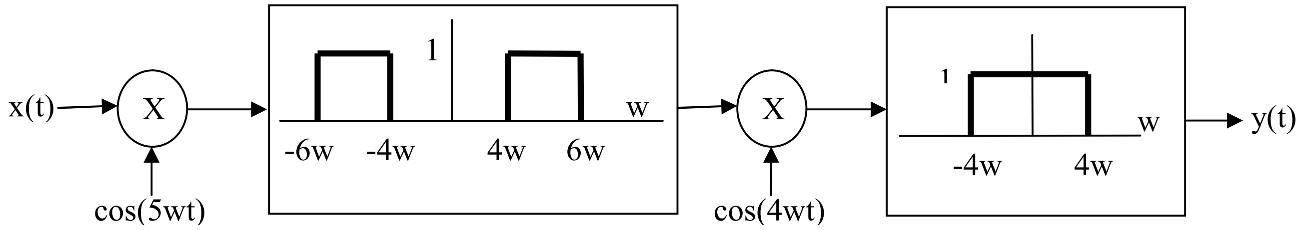
X(jw) after 2nd modulation (5w)

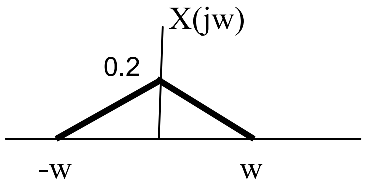


X(jw) after 2nd Low Pass Filter



7U. Determine and sketch Y(jw), the frequency spectrum of y(t) for the following system. Where X(jw) is the Fourier Transform of x(t):

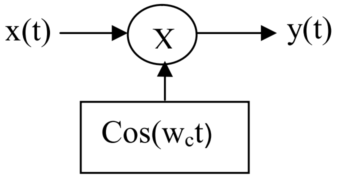




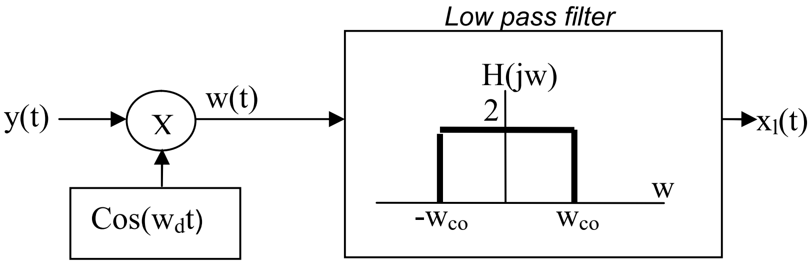
Solution:

8S. $x(t)$ is band limited $\{X(jw) = 0$ for $|w| \geq w_M\}$. Consider an amplitude modulation and demodulation pair where each have a different frequencies (w_c and w_d) as shown below:

Modulation system
$y(t) = x(t)\cos(w_c t)$



Demodulation system
$w(t) = y(t)\cos(w_d t)$



Low pass filter cut off frequency, w_{co}, is constrained as shown below:

$$w_M + \Delta w < w_{co} < 2w_c + \Delta w - w_M \qquad \textit{Where} \quad \Delta w = w_d - w_c$$

For the above system:

a) Show that the low pass filter output, $x_l(t)$, is proportional to $x(t)\cos(\Delta w t)$.

b) Sketch the spectrum of the output of the demodulator.

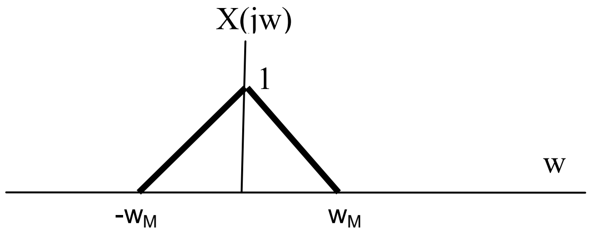
Solution:

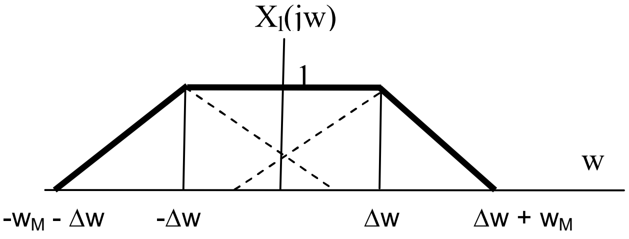
a)
$w(t) = x(t)\cos(w_d t)\cos(w_c t)$
Apply Trig. Relation → $\cos(a)\cos(b) = \cos(a+b) + \cos(a-b)$
$w(t) = \frac{1}{2} x(t)\{\cos(w_d + w_c)t + \cos(w_d - w_c)t\}$
$w(t) = \frac{1}{2} x(t)\{\cos(2w_c + \Delta w)t + \cos(\Delta w)t\}$
$w(t) = \frac{1}{2} x(t)\cos(2w_c + \Delta w)t + \frac{1}{2} x(t)\cos(\Delta w)t\}$

First term frequency spectrum → $2w_c + \Delta w - w_M \leq |w_1| \leq 2w_c + \Delta w + w_M$ so this term will not pass through the low pass filter therefore the low pass output is:

$x(t) = x(t)\cos(\Delta w)t$

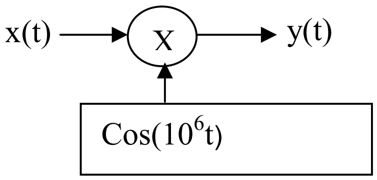
b)



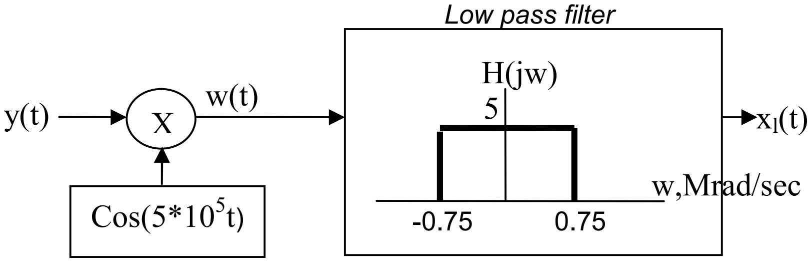


8U. x(t) is band limited {X(jw) = 0 for |w| ≥ 5000 }. Consider an amplitude modulation and demodulation pair where each have a different frequencies as shown below:

Modulation system



Demodulation system



For the above system:

a) Show that the low pass filter output, $x_l(t)$, is proportional to $x(t)\cos(\Delta w t)$.

b) Sketch the spectrum of the output of the demodulator.

Solution: